What do experimental data tell us?

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Conjectures on the form of economic and natural low-energy SUSY

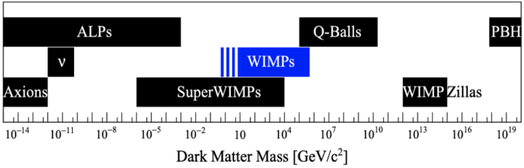
Past insights: MSSM may be incomplete (2007);
SUSY was going to face severe challenges (2015).

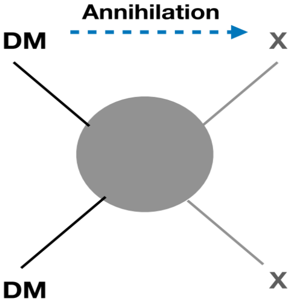
1. Preliminaries of WIMP DM **(from Jia Liu's talk)**
2. Criteria in Estimating the Goodness of a Theory
3. Example I: MSSM
4. Example II: Z_3-NMSSM
5. Example III: General NMSSM
6. Example IV: Type-I NMSSM
7. Example V: B-L NMSSM
8. Conclusions

Part One

Preliminaries of WIMP DM

Preliminary: the freeze-out of thermal DM

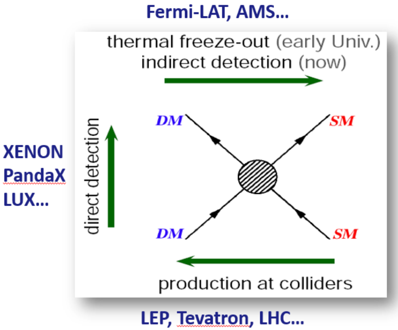




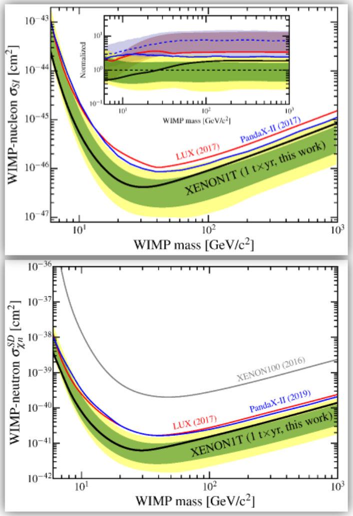
X:SM or non-SM particle

- Mass bound: $N_{\rm eff}$ from CMB, unitary
  $5\ \text{MeV} \lesssim \text{m}_{\rm DM} \lesssim 110\text{TeV}$;
- DM starts with thermal distribution;
- Relic abundance is determined by freeze-out mechanism;
- DM has an electroweak-scale coupling (WIMP miracle).
  Consider DM DM $\to$ X X:
  $\langle \sigma v \rangle \sim \frac{g^4}{m_{\rm DM}^2} \sim 3 \times 10^{-26}\ \text{cm}^3\ \text{s}^{-1} \Rightarrow g \sim \sqrt{\frac{m_{\rm DM}}{10\text{TeV}}}$,
  $g \sim 0.1$ for $m_{\rm DM} = 100$ GeV.

Preliminary: limits from direct detection experiments



No new physics signal !



arXiv:1805.12562

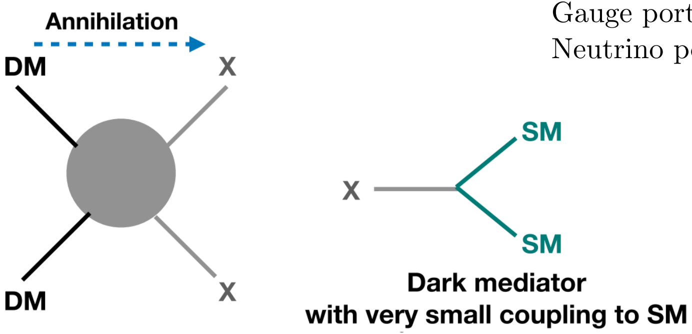
arXiv:1902.03234

Preliminary PandaX-4T results released!

Preliminary: The way-out from direct detection limits

- 1. Very small coupling:
  - 1.1 Secluded dark matter (dark sector)

Proposed in 2007.
Three types of portals:
Higgs portal;
Gauge portal;
Neutrino portal



This mechanism can be realized in non-minimal SUSY!

Preliminary: The way-out from direct detection limits

- 2. Suppressed scattering cross-section:

  - By velocity or momentum transfer

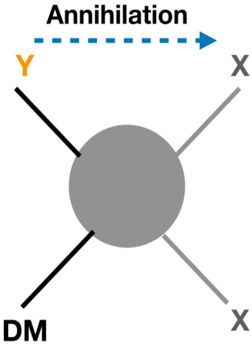
Case for Fermionic DM

Kumar & Marfatia:1305.1611 (PRD)

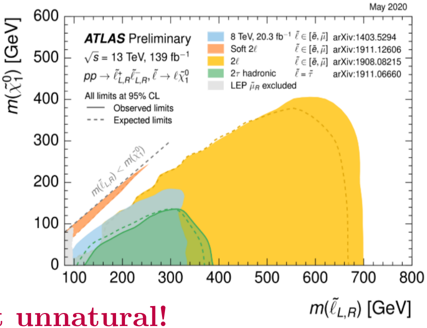
	Name	Interaction Structure	$\sigma_{\rm SI}$ suppression	$\sigma_{\rm SD}$ suppression	s-wave?
Scalar	F1	$\bar{X}X\bar{q}q$	1	$q^2 v^{\perp 2}$ (SM)	No
	F2	$\bar{X}\gamma^5 X\bar{q}q$	q^2 (DM)	$q^2 v^{\perp 2}$ (SM); q^2 (DM)	Yes
	F3	$\bar{X}X\bar{q}\gamma^5 q$	0	q^2 (SM)	No
Pseudoscalar	F4	$\bar{X}\gamma^5 X\bar{q}\gamma^5 q$	0	q^2 (SM); q^2 (DM)	Yes
Vector	F5	$\bar{X}\gamma^\mu X\bar{q}\gamma_\mu q$ (vanishes for Majorana X)	1	$q^2 v^{\perp 2}$ (SM) q^2 (SM); q^2 or $v^{\perp 2}$ (DM)	Yes
Anapole	F6	$\bar{X}\gamma^\mu\gamma^5 X\bar{q}\gamma_\mu q$	$v^{\perp 2}$ (SM or DM)	q^2 (SM)	No
	F7	$\bar{X}\gamma^\mu X\bar{q}\gamma_\mu\gamma^5 q$ (vanishes for Majorana X)	$q^2 v^{\perp 2}$ (SM); q^2 (DM)	$v^{\perp 2}$ (SM) $v^{\perp 2}$ or q^2 (DM)	Yes
	F8	$\bar{X}\gamma^\mu\gamma^5 X\bar{q}\gamma_\mu\gamma^5 q$	$q^2 v^{\perp 2}$ (SM)	1	$\propto m_f^2/m_X^2$
	F9	$\bar{X}\sigma^{\mu\nu} X\bar{q}\sigma_{\mu\nu} q$ (vanishes for Majorana X)	q^2 (SM); q^2 or $v^{\perp 2}$ (DM) $q^2 v^{\perp 2}$ (SM)	1	Yes
	F10	$\bar{X}\sigma^{\mu\nu}\gamma^5 X\bar{q}\sigma_{\mu\nu} q$ (vanishes for Majorana X)	q^2 (SM)	$v^{\perp 2}$ (SM) q^2 or $v^{\perp 2}$ (DM)	Yes

Not easy to build specific models! Let alone in supersymmetric

- 3. Coannihilation mechanism



- Y has a close mass with DM
  - Y is not populated today due to decay
  - Charged Y: near degenerate spectrum of SUSY, AMSB; EW multiplet DM (2n+1, 0) ($\delta m \sim$ 166 MeV)



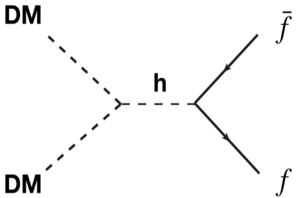
Easily realized in SUSY, but unnatural!

- 4. Resonant annihilation

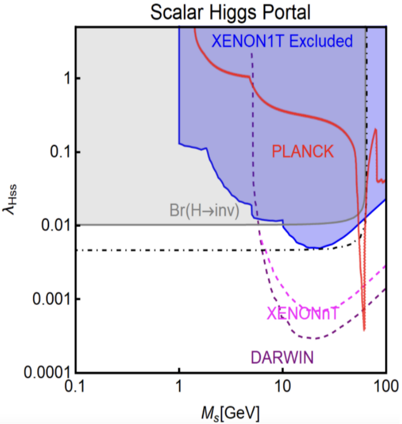
- $2m_{\rm DM} \approx m_X$

Scalar DM (s) with a Higgs portal coupling

$$\Delta\mathcal{L}_s = -\frac{1}{2}m_s^2 s^2 - \frac{1}{4}\lambda_s s^4 - \frac{1}{4}\lambda_{Hss}\phi^\dagger\phi s^2$$



+ 2 diagrams to hh



Arcadi et al: 2101.02507

See also WL Guo, LY Wu et al 2010; B Li, YF Zhou 2015

Easily realized in SUSY, but needs severe fine-tuning!

Preliminary: The way-out from direct detection limits

- 5. Cancellation effect in scattering cross-section

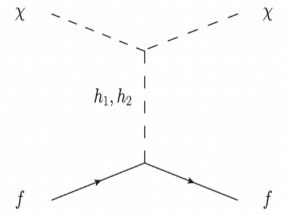
  - SM Higgs - Dark scalar mediator cancellation

Gross, Lebedev1, Toma: 1708.02253 (PRL)

$$V_0 = -\frac{\mu_H^2}{2}|H|^2 - \frac{\mu_S^2}{2}|S|^2 + \frac{\lambda_H}{2}|H|^4 + \lambda_{HS}|H|^2|S|^2 + \frac{\lambda_S}{2}|S|^4$$

$$V_{\rm soft} = -\frac{\mu_S'^2}{4}S^2 + \text{h.c.} \qquad \text{symmetry}: S \leftrightarrow S^*$$

$$S = (v_s + s + i\chi)/\sqrt{2}$$ Pseudoscalar DM



CP-even scalar mixing (s, h) $\rightarrow (h_1, h_2)$

$$\mathcal{L} \supset -(h_1\cos\theta + h_2\sin\theta)\sum_f \frac{m_f}{v}\bar{f}f \qquad \mathscr{L} \supset \frac{\chi^2}{2v_s}\left(m_{h_1}^2 \sin\theta h_1 - m_{h_2}^2\cos\theta h_2\right)$$

$$\mathcal{A}_{dd}(t) \propto \sin\theta\cos\theta\left(\frac{m_{h_2}^2}{t-m_{h_2}^2} - \frac{m_{h_1}^2}{t-m_{h_1}^2}\right) \simeq \sin\theta\cos\theta\,\frac{t\,(m_{h_2}^2 - m_{h_1}^2)}{m_{h_1}^2\, m_{h_2}^2} \simeq 0$$

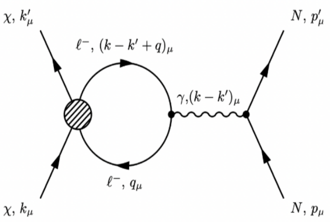
See JL, XP Wang and F Yu 1704.00730 (JHEP), for cancellation between A' - Z boson in kinetic mixing dark photon model

The amplitude is suppressed by q^2

Can not be realized in SUSY!

Preliminary: The way-out from direct detection limits

- 6. Leptophilic models
  - Only couples to electrons, couples to nucleons at 1-loop
    - For light DM, e-DM recoils can have stringent limits (e.g. XENON1T, PANDAX, CDEX)
    - For heavy DM, neucleus-DM recoils wins over e-DM recoil



$$R^{\mathrm{WAS}} : R^{\mathrm{WES}} : R^{\mathrm{WNS}} \sim \epsilon_{\mathrm{WAS}} : \epsilon_{\mathrm{WES}} \frac{m_e}{m_N} : \left(\frac{\alpha_{\mathrm{em}} Z}{\pi}\right)^2 \sim 10^{-17} : 10^{-10} : 1$$

WAS = e kicked out

WES = e to higher energy level

WNS = nucleus recoil

The probability to find a high p electron in the wave function is highly suppressed!

Kopp et al: 0907.3159 (PRD)

Realized the SM extensionos with $L_{\mu-\tau}$, $B-L$, or left-right symmetry, and their supersymmetric versions!

Preliminary: indirect detection from DM annihilation

- observable quantity:
  CMB photon, photon from dwarf galaxies, and positron from cosmic ray, etc.
- $S \propto \sum_i \langle \sigma v \rangle_{0,i} \epsilon_i$,
  $\langle \sigma v \rangle_{0,i}$: annihilation rate for DM DM $\to \mathrm{e^+e^-}, \mu^+\mu^-, \tau^+\tau^-, \mathrm{t\bar{t}}, \cdots$ at present day;
  ϵ_i: efficiency translating annihilation products into signal.

Note: $\langle \sigma v \rangle_F$ may differ significantly from $\langle \sigma v \rangle_{0,i}$

$\sigma v \sim \sigma_s + \sigma_p v^2 + \sigma_d v^4 + \ldots$ (s-, p-, and d-wave contribution)

- Freeze-out: $v^2 \sim 0.25$
- CMB: $v^2 \sim \mathrm{eV}/m_{\mathrm{DM}} \sim 10^{-5}$
- Today: $v \sim 10^{-3} c$

ϵ_i **may differ greatly for different annihilation final state!**

Preliminary: indirect detection from DM annihilation

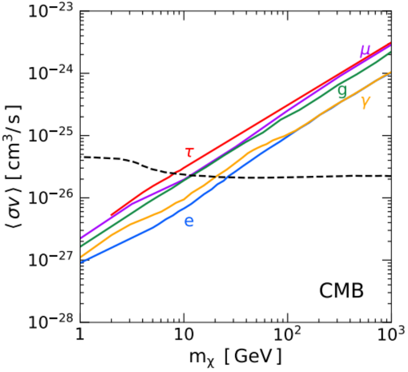


Figure 1: Planck CMB limits at 95% C.L. for DM annihilation 100% to individual channels.

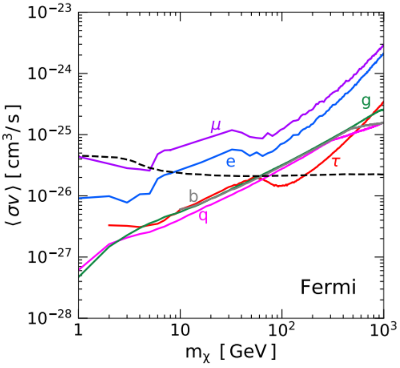


Figure 2: Fermi-LAT limits at 95% C.L. for DM annihilation 100% to individual channels.

Depending on final state, CMB limits are powerful for light DM, while Fermi-LAT limits are effective for $m_{\rm DM} \lesssim 100$ GeV.

Preliminary: indirect detection from DM annihilation

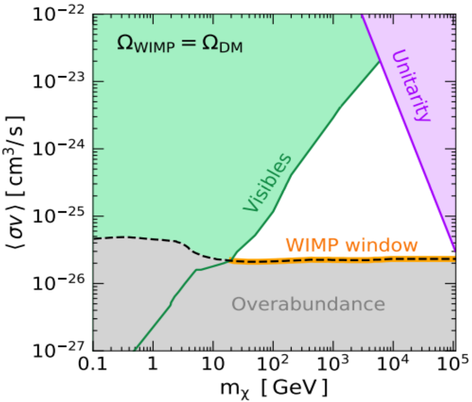


Figure 3: Bounds on the generic thermal WIMP window (s-wave $2 \to 2$ annihilation, standard cosmological history), assuming WIMP DM is 100% of the DM. Shown is the conservative bound calculated from the data of CMB, Fermi-LAT and AMS-02 (Visibles), and the unitarity bound. The remaining WIMP window is the orange line, and the white space is unprobed. Thermal relic cross section is the dashed line.

Conclusion about WIMP DM

1. As far as WIMP DM itself is concerned, it can fit experiments very well.
2. WIMP DM can be easily embedded into renormalizable theories. In this case, DM physics usually entangles with Higgs physics, sparticle physics, and sometimes neutrino physics. Global fit is necessary.
3. In economic WIMP DM theories, DM physics are usually in tension with various experiments, and consequently, the theories become unnatural.
4. What is the most economic and natural (supersymmetric) WIMP DM theory?

Choose eigenvalues as inputs can simplify calculation.

Neutralino mass matrix in the MSSM:

$$M_N = \begin{pmatrix} M_1 & 0 & -\frac{v g_1 c_\beta}{2} & \frac{v g_1 s_\beta}{2} \\ 0 & M_2 & \frac{v c_W g_1 c_\beta}{2 s_W} & -\frac{v c_W g_1 s_\beta}{2 s_W} \\ -\frac{v g_1 c_\beta}{2} & \frac{v c_W g_1 c_\beta}{2 s_W} & 0 & -\mu \\ \frac{v g_1 s_\beta}{2} & -\frac{v c_W g_1 s_\beta}{2 s_W} & -\mu & 0 \end{pmatrix}$$

$$N_{i,j} = \frac{1}{\sqrt{C_i}} \begin{pmatrix} \left(\mu^2 - m_{\chi_i}^2\right)\left(M_2 - m_{\chi_i}\right) - M_Z^2 c_W^2 \left(m_{\chi_i} + 2\mu s_\beta c_\beta\right) \\ -M_Z^2 s_W c_W \left(m_{\chi_i} + 2\mu s_\beta c_\beta\right) \\ \left(M_2 - m_{\chi_i}\right)\left(m_{\chi_i} c_\beta + \mu s_\beta\right) M_Z s_W \\ -\left(M_2 - m_{\chi_i}\right)\left(m_{\chi_i} s_\beta + \mu c_\beta\right) M_Z s_W \end{pmatrix}_j$$

C_i: Normalization factor; **Other technique: mass insertion**.

$$C_i = M_Z^2 c_W^2 \left(m_{\chi_i} + 2\mu s_\beta c_\beta\right)\left[M_Z^2 \left(m_{\chi_i} + 2\mu s_\beta c_\beta\right) + 2\left(\mu^2 - m_{\chi_i}^2\right)\left(m_{\chi_i} - M_2\right)\right] + \left(m_{\chi_i} - M_2\right)^2 \left\{M_Z^2 s_W^2 \left[\left(m_{\chi_i}^2 + \mu^2\right) + 4\mu m_{\chi_i} s_\beta c_\beta\right] + \left(m_{\chi_i}^2 - \mu^2\right)^2\right\}$$

Preliminary: How to study SUSY phenomenology?

Many many ways! Most advanced method:
Fit theory to experimental data, extract underlying physics!

1. Construct likelihood function from experimental data;
2. Scan theory's parameter space with advanced algorithm:
   - Characteristics of the parameter space:
     high dimensional, highly degenerated likelihood, isolated physical parameter island, inefficient for random and Markov chain scan;
   - MultiNest algorithm is well adaptive for such a situation:
     use *nlive* samples to decide iso-likelihood contour in each iteration;
     provide comprehensive information of the space;
     the results are statistically significant. **Why?** $\Longrightarrow$
3. Scrutinize the properties of obtained parameter points, e.g., prediction on various experimental measurements, theoretical fine-tuning, vacuum stability, etc.. **Human learning materials!**
4. Analyze global features of the theory by statistics:
   Some fundamental physical mechanisms can be inferred.
5. Provide intuitive understandings with the help of analyt. formulae.

Preliminary: Statistics-Bayesian theorem

Bayesian theorem: $\Theta = (\Theta_1, \Theta_2, \cdots)$ is theoretical input parameters.

$$\mathrm{P}(\boldsymbol{\Theta} \mid \mathbf{D}, H) \equiv \frac{\mathrm{P}(\mathbf{D} \mid \boldsymbol{\Theta}, H)\, \mathrm{P}(\boldsymbol{\Theta} \mid H)}{\mathrm{P}(\mathbf{D} \mid H)} \Longrightarrow P(\boldsymbol{\Theta}) \equiv \frac{\mathcal{L}(\boldsymbol{\Theta})\pi(\boldsymbol{\Theta})}{\mathcal{Z}}$$

- $\mathrm{P}(\boldsymbol{\Theta} \mid \mathbf{D}, H) \equiv P(\boldsymbol{\Theta})$: Posterior probability distribution function.
- $\mathrm{P}(\mathbf{D} \mid \boldsymbol{\Theta}, H) \equiv \mathcal{L}(\boldsymbol{\Theta})$: Likelihood function.
- $\mathrm{P}(\boldsymbol{\Theta} \mid H) \equiv \pi(\boldsymbol{\Theta})$: Prior probability Density Function.
- $\mathrm{P}(\mathbf{D} \mid H) \equiv \mathcal{Z}$: Bayesian evidence, normalization factor.

$P(\Theta)$: the state of our knowledge about the parameters Θ given the experimental data D, or alternatively speaking, the updated prior PDF after considering the impact of the experimental data.
One can infer from $P(\Theta)$ the underlying physics of the model.

Preliminary: Statistics-Bayesian theorem

Likelihood function: the preference of experimental results to parameter point $p = \{\Theta\}$, e.g., Gaussian distribution:

$$\mathcal{L} = e^{-\frac{[\mathcal{O}_{th}(\Theta)-\mathcal{O}_{exp}]^2}{2\sigma^2}}.$$

$\mathcal{O}_{th}(\Theta)$: theoretical prediction, $\mathcal{O}_{exp}$: experimental measurement, σ: total uncertainty.

Bayesian evidence: averaged likelihood, reflecting theory's capability to keep consistent with the data.

$$\mathcal{Z} = \int \mathcal{L}(\mathbf{\Theta})\pi(\mathbf{\Theta})d^D\mathbf{\Theta}.$$

D: Dimension of the parameter space.

Preliminary: Statistics-Bayesian theorem

Marginal posterior PDFs: reflecting the preference to specific regions of one or more parameters.

$$1D: P\left(\Theta_A\right)=\int P(\Theta) d\Theta_1 d\Theta_2 \cdots d\Theta_{A-1} d\Theta_{A+1} \cdots \cdots$$

$$2D: P\left(\Theta_A, \Theta_B\right)=\int P(\Theta) d\Theta_1 d\Theta_2 \cdots d\Theta_{A-1} d\Theta_{A+1} \cdots d\Theta_{B-1} d\Theta_{B+1} \cdots$$

Credible Regions: most preferred parameter regions by data; it depends on both likelihood function and phase space.

$$1D: \int_{\Theta_{A_1}}^{\Theta_{A_2}} P\left(\Theta_A\right) d\Theta_A=1-\alpha$$

$$2D: \int_{P\left(\Theta_A, \Theta_B\right) \geq p_{\text{crit}}} P\left(\Theta_A, \Theta_B\right) d\Theta_A d\Theta_B=1-\alpha$$

$$1\sigma:\ \alpha=0.317, \qquad 2\sigma:\ \alpha=0.055.$$

Preliminary: Statistics-Frequentists

Profile Likelihood: parameter's capability to explain the data.

$$1D : \mathcal{L}\left(\Theta_A\right) = \max_{\Theta_1,\cdots,\Theta_{A-1},\Theta_{A+1},\cdots} \mathcal{L}(\Theta),$$

$$2D : \mathcal{L}\left(\Theta_A, \Theta_B\right) = \max_{\Theta_1,\cdots,\Theta_{A-1},\Theta_{A+1},\cdots,\Theta_{B-1},\Theta_{B+1},\cdots} \mathcal{L}(\Theta)$$

Confidence Intervals: most favored regions to explain the data; it depends only on the likelihood function.

$$1D : \left\{\chi^2(\Theta_A) - \chi^2_{Best}\right\} \leq F^{-1}_{\chi^2_1}(1-\alpha),$$

$$2D : \left\{\chi^2(\Theta_A, \Theta_B) - \chi^2_{Best}\right\} \leq F^{-1}_{\chi^2_2}(1-\alpha)$$

$\chi^2\left(\Theta_A\right) \equiv -2\log\mathcal{L}\left(\Theta_A\right), \quad \chi^2\left(\Theta_A, \Theta_B\right) \equiv -2\log\mathcal{L}\left(\Theta_A, \Theta_B\right);$

χ^2_{Best}: the χ^2 value for the best point;

$F^{-1}_{\chi^2_n}$: the inverse cdf for a chi-squared distribution with n dof:

1σ $(\alpha = 0.317)$: $F^{-1}_{\chi^2_1} = 1.00, \quad F^{-1}_{\chi^2_2} = 2.30;$

2σ $(\alpha = 0.046)$: $F^{-1}_{\chi^2_1} = 4.00, \quad F^{-1}_{\chi^2_2} = 6.18.$

Part Two

Criteria in Estimating the Goodness of a Theory

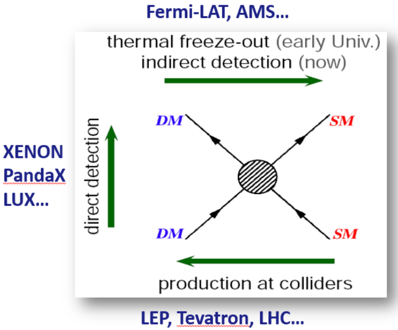
CEGT: Available Experimental Data

Rich experimental data have been accumulated !

- Precision electroweak data;
- Heavy flavor data;
- Neutrino experiments;
- Higgs property measurement.
- Dark matter search experiments;
- LHC search for supersymmetry;
- Muon anomalous magnetic moment.

Global Fit: Combine all the data to analyze theories.

CEGT: WIMP DM direct search experiments



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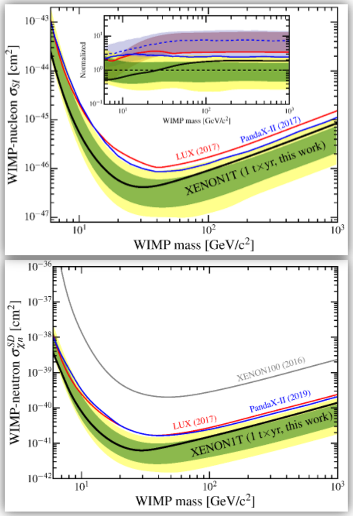


arXiv:1805.12562

arXiv:1902.03234

Preliminary PandaX-4T results released!

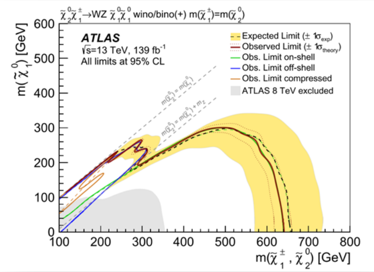
CEGT: Implications of DM search experiments

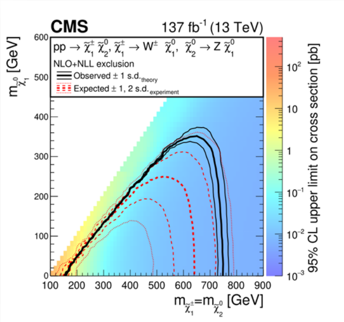
Popular WIMP DM candidate: Bino-dominated $\tilde{\chi}_1^0$ in MSSM. DM-nucleon scatterings proceed by t-channel exchange of Higgs and Z boson, respectively.

$$\sigma^{SI}_{\tilde{\chi}_1^0-N} \simeq 5\times 10^{-45}\ \mathrm{cm}^2 \left(\frac{\mathrm{C}_{\tilde{\chi}_1^0\tilde{\chi}_1^0\mathrm{h}}}{0.1}\right)^2 \left(\frac{\mathrm{m_h}}{125\mathrm{GeV}}\right)^2$$

$$\sigma^{SD}_{\tilde{\chi}_1^0-N} \simeq 10^{-39}\ \mathrm{cm}^2 \left(\frac{\mathrm{C}_{\tilde{\chi}_1^0\tilde{\chi}_1^0\mathrm{Z}}}{0.1}\right)^2$$

1. Interactions of DM with SM particles are **feeble at most** when $m_{\tilde{\chi}_1^0} \sim 100\mathrm{GeV}$.
2. Difficult to obtain the measured abundance if DM DM $\to$ SM SM. Exceptions: Co-annihilation, Resonance annihilation. **All corresponds to a small Bayesian evidence, fine-tunned!**
3. **Simple WIMP DM theories are becoming unnatural!**
4. Good theory: Naturally explaining the experimental results. E.g., secluded DM theories in a more complex framework.

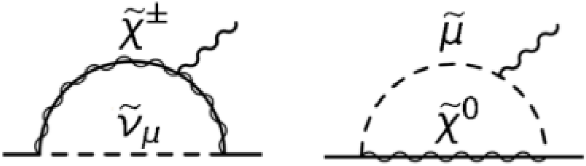




Latest LHC searches for tri- and bi-lepton signals.

1. Simplified model for a specified process.
2. Invalid for a specific theory: complex decay chain, multiple production processes, and various signals to be analyzed.
3. Elaborated Monte Carlo simulations are necessary.

Muon g − 2 in SUSY:



Operator contributing to a_μ: $\frac{a_\mu}{m_\mu}\ \bar{b}\sigma_{\mu\nu}bF^{\mu\nu}$.
Note that the operator involves the chiral flipping of μ leptons.

Muon g − 2 in SUSY: neglecting neutrino Yukawa couplings

$$a_\mu^{\rm SUSY} = a_\mu^{\tilde{\chi}^0\tilde{\mu}} + a_\mu^{\tilde{\chi}^\pm\tilde{\nu}}$$

$$a_\mu^{\tilde{\chi}^0\tilde{\mu}} = \frac{m_\mu}{16\pi^2}\sum_{i,l}\left\{-\frac{m_\mu}{12m^2_{\tilde{\mu}_l}}\left(\left|n^{\rm L}_{il}\right|^2+\left|n^{\rm R}_{il}\right|^2\right)F_1^{\rm N}(x_{il}) + \frac{m_{\tilde{\chi}^0_i}}{3m^2_{\tilde{\mu}_l}}{\rm Re}\left(n^{\rm L}_{il}n^{\rm R}_{il}\right)F_2^{\rm N}(x_{il})\right\}$$

$$a_\mu^{\tilde{\chi}^\pm\tilde{\nu}} = \frac{m_\mu}{16\pi^2}\sum_{k}\left\{\frac{m_\mu}{12m^2_{\tilde{\nu}_\mu}}\left(\left|c^{\rm L}_{k}\right|^2+\left|c^{\rm R}_{k}\right|^2\right)F_1^{\rm C}(x_{k}) + \frac{2m_{\tilde{\chi}^\pm_k}}{3m^2_{\tilde{\nu}_\mu}}{\rm Re}\left(c^{\rm L}_{k}c^{\rm R}_{k}\right)F_2^{\rm C}(x_{k})\right\}$$

$i = 1, \cdots, 5$: neutralino index, $k = 1, 2$: chargino index, $l = 1, 2$: smuon index.

$$n^{\rm L}_{il} = \frac{1}{\sqrt{2}}\left(g_2 N_{i2} + g_1 N_{i1}\right) X^*_{l1} - y_\mu N_{i3} X^*_{l2}, \quad n^{\rm R}_{il} = \sqrt{2} g_1 N_{i1} X_{l2} + y_\mu N_{i3} X_{l1},$$

$$c^{\rm L}_k = -g_2 V^{\rm c}_{k1}, \quad c^{\rm R}_k = y_\mu U^{\rm c}_{k2}$$

N, X, U^c, V^c: the neutralino, smuon and chargino mass rotation matrices, $U^{\rm c*} X_{2\times 2} V^{\rm c\dagger} = m^{\rm diag}_{\tilde{\chi}^\pm}$.

CEGT: Improved measurement of Muon g-2

$x_{il} \equiv m^2_{\tilde{\chi}^0_i}/m^2_{\tilde{\mu}_l}$, $x_k \equiv m^2_{\tilde{\chi}^\pm_k}/m^2_{\tilde{\nu}_\mu}$

$$F_1^N(x) = \frac{2}{(1-x)^4}\left[1 - 6x + 3x^2 + 2x^3 - 6x^2 \ln x\right]$$
$$F_2^N(x) = \frac{3}{(1-x)^3}\left[1 - x^2 + 2x \ln x\right]$$
$$F_1^C(x) = \frac{2}{(1-x)^4}\left[2 + 3x - 6x^2 + x^3 + 6x \ln x\right]$$
$$F_2^C(x) = -\frac{3}{2(1-x)^3}\left[3 - 4x + x^2 + 2 \ln x\right]$$

$F_1^N(1) = F_2^N(1) = F_1^C(1) = F_2^C(1) = 1$ for mass-degenerate sparticle case.

CEGT: Improved measurement of Muon g-2

Mass insertion method: **WHL**, **WHR**, **BHR**, and **BLR** diagrams contribute to $a_\mu^{\rm SUSY}$.

$$a^{\rm SUSY}_{\mu,{\rm WHL}} = \frac{\alpha_2}{8\pi}\frac{m_\mu^2 \mu M_2 \tan\beta}{m^4_{\tilde{\nu}_\mu}}\left\{2 f_C\left(\frac{M_2^2}{m^4_{\tilde{\nu}_\mu}}, \frac{\mu^2}{m^2_{\tilde{\nu}_\mu}}\right) - \frac{m^4_{\tilde{\nu}_\mu}}{m^4_{\tilde{\mu}_L}} f_N\left(\frac{M_2^2}{m^2_{\tilde{\mu}_L}}, \frac{\mu^2}{m^2_{\tilde{\mu}_L}}\right)\right\}$$

$$a^{\rm SUSY}_{\mu,{\rm BHL}} = \frac{\alpha_Y}{8\pi}\frac{m_\mu^2 \mu M_1 \tan\beta}{m^4_{\tilde{\mu}_L}} f_N\left(\frac{M_1^2}{m^2_{\tilde{\mu}_L}}, \frac{\mu^2}{m^2_{\tilde{\mu}_L}}\right)$$

$$a^{\rm SUSY}_{\mu,{\rm BHR}} = -\frac{\alpha_Y}{4\pi}\frac{m_\mu^2 \mu M_1 \tan\beta}{m^4_{\tilde{\mu}_R}} f_N\left(\frac{M_1^2}{m^2_{\tilde{\mu}_R}}, \frac{\mu^2}{m^2_{\tilde{\mu}_R}}\right)$$

$$a^{\rm SUSY}_{\mu,{\rm BLR}} = \frac{\alpha_Y}{4\pi}\frac{m_\mu^2 \mu M_1 \tan\beta}{M_1^4} f_N\left(\frac{m^2_{\tilde{\mu}_L}}{M_1^2}, \frac{m^2_{\tilde{\mu}_R}}{M_1^2}\right)$$

where the loop functions are given by:

$$f_C(x,y) = \frac{5-3(x+y)+xy}{(x-1)^2(y-1)^2} - \frac{2\ln x}{(x-y)(x-1)^3} + \frac{2\ln y}{(x-y)(y-1)^3}$$

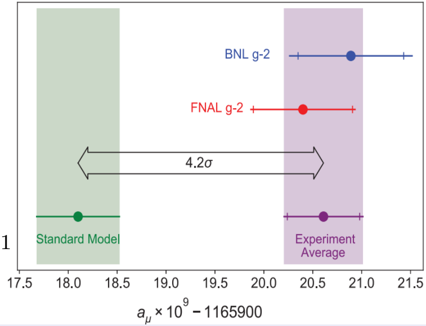
$$f_N(x,y) = \frac{-3+x+y+xy}{(x-1)^2(y-1)^2} + \frac{2x\ln x}{(x-y)(x-1)^3} - \frac{2y\ln y}{(x-y)(y-1)^3}$$

$$f_C(1,1) = 1/2,\ f_N(1,1) = 1/6$$

CEGT: Improved measurement of Muon g-2

After considering experimental constraints on sparticle masses, mass insertion method is a very good approximation.

$$\Delta a_\mu = a_\mu^{\rm Exp} - a_\mu^{\rm SM} = (251 \pm 59) \times 10^{-11}$$



1. Moderately large $\tan\beta$ is necessary!
2. Set an upper bound on LSP mass (about 600 GeV).
3. Set an upper bound on NLSP mass (about 700 GeV).
4. The other involved sparticles cannot be excessively heavy.
5. LHC search tightly limits SUSY explanantions: $1 + 1 \gg 2$.
6. Global fits with/without Muon g-2 differes significantly.

CEGT: Critical aspects of TeV-scale SUSY

Readily explain data, particularly those for correlated obs.!

1. **DM physics:** Ωh^2 versus $\sigma_{\tilde{\chi}_1^0-p}$ / Bayesian Evidence.
   $\times/\checkmark$: explain the experimental results with/without tuning.
2. **LHC and Δa_μ:** SUSY searches versus sizable correction to a_μ.
   $\times/\checkmark$: tight/loose constraints on the explanation of Muon g-2.
3. **Natural EWSB:** $m_Z^2 = 2(m_{H_d}^2 - m_{H_u}^2 \tan^2\beta)/(\tan^2\beta - 1) - \mathbf{2\mu^2}$.
   $\times/\checkmark$: whether or not a moderately small μ is preferred.
4. **Neutrino physics:** neutrino masses and mixings.
   $\times/\checkmark$: whether or not providing reasonable mechanisms for ....
5. **Higgs physics:** unreasonably large mass, SM-like couplings
   $\times/\checkmark$: explain the mass with/without large radiative corrections.

CEGT: Status of different supersymmetric theories

Obtained by both analytic formulae and global fits, which are tough tasks.

Model	MSSM	Z_3-NMSSM		GNMSSM	Type-I NMSSM	B-L NMSSM
DM component	Bino	Bino	Singlino	Singlino	Sneutrino	Bino, Singlino, BLino, Bileptino, or sneutrino
DM physics	×	×	×	✓	✓✗	✓
LHC and Δa_μ	×	×	×	✓	✓	✓
EWSB	×	×	✓	✓	✓	✓
Neutrino	×	×	×	×	✓✗	✓
Higgs mass	×	×	×	×	✓✗	✓

Part Three

Example I: MSSM

MSSM

- Vector Superfields

SF	Spin $\frac{1}{2}$	Spin 1	$SU(N)$	Coupling	Name
$\hat{B}$	$\lambda_{\tilde{B}}$	B	U(1)	g_1	hypercharge
$\hat{W}$	$\lambda_{\tilde{W}}$	W	SU(2)	g_2	left
$\hat{g}$	$\lambda_{\tilde{g}}$	g	SU(3)	g_3	color

- Chiral Superfields

SF	Spin 0	Spin $\frac{1}{2}$	Generations	(U(1) $\otimes$ SU(2) $\otimes$ SU(3))
$\hat{q}$	$\tilde{q}$	q	3	$(\frac{1}{6}, \mathbf{2}, \mathbf{3})$
$\hat{l}$	$\tilde{l}$	l	3	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{H}_d$	H_d	$\tilde{H}_d$	1	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{H}_u$	H_u	$\tilde{H}_u$	1	$(\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{d}$	$\tilde{d}_R^*$	d_R^*	3	$(\frac{1}{3}, \mathbf{1}, \overline{\mathbf{3}})$
$\hat{u}$	$\tilde{u}_R^*$	u_R^*	3	$(-\frac{2}{3}, \mathbf{1}, \overline{\mathbf{3}})$
$\hat{e}$	$\tilde{e}_R^*$	e_R^*	3	$(1, \mathbf{1}, \mathbf{1})$

- Superpotential: μ-the only dimensional parameter. μ-problem!

$$W_{\rm MSSM} = \mu \hat{H}_u \hat{H}_d - Y_d \hat{d} \hat{q} \hat{H}_d - Y_e \hat{e} \hat{l} \hat{H}_d + Y_u \hat{u} \hat{q} \hat{H}_u$$

MSSM

- Neutralino mass matrix

$$m_{\tilde{\chi}^0} = \begin{pmatrix} M_1 & 0 & -\frac{1}{2}g_1 v_d & \frac{1}{2}g_1 v_u \\ 0 & M_2 & \frac{1}{2}g_2 v_d & -\frac{1}{2}g_2 v_u \\ -\frac{1}{2}g_1 v_d & \frac{1}{2}g_2 v_d & 0 & -\mu \\ \frac{1}{2}g_1 v_u & -\frac{1}{2}g_2 v_u & -\mu & 0 \end{pmatrix}$$

DM is Bino-dominated and its couplings are given by:

$$C_{\tilde{\chi}_1^0 \tilde{\chi}_1^0 h} \simeq e \tan\theta_W \frac{m_Z}{\mu\left(1 - m_{\tilde{\chi}_1^0}^2/\mu^2\right)} \left(\sin 2\beta + \frac{m_{\tilde{\chi}_1^0}}{\mu}\right)$$

$$C_{\tilde{\chi}_1^0 \tilde{\chi}_1^0 Z} \simeq \frac{e \tan\theta_W \cos 2\beta}{2} \frac{m_Z^2}{\mu^2 - m_{\tilde{\chi}_1^0}^2}$$

The expressions are valid only by assuming that $|\mu| \gg |m_{\tilde{\chi}_1^0}|$.
DM direct detection experiments prefer $\mu > 300$ GeV.
Blind spot: $\sin 2\beta + m_{\tilde{\chi}_1^0}/\mu = 0$; Enhancement: $|m_{\tilde{\chi}_1^0}/\mu| \simeq 1$.

MSSM: more explanations

Popular WIMP DM candidate in MSSM: Bino-dominated $\tilde{\chi}_1^0$

$$\sigma^{SI}_{\tilde{\chi}_1^0-N} \simeq 5\times 10^{-45}\ \mathrm{cm}^2 \left(\frac{\mathrm{C}_{\tilde{\chi}_1^0\tilde{\chi}_1^0\mathrm{h}}}{0.1}\right)^2 \left(\frac{\mathrm{m_h}}{125\mathrm{GeV}}\right)^2$$

$$\sigma^{SD}_{\tilde{\chi}_1^0-N} \simeq 10^{-39}\ \mathrm{cm}^2 \left(\frac{\mathrm{C}_{\tilde{\chi}_1^0\tilde{\chi}_1^0\mathrm{Z}}}{0.1}\right)^2$$

Note $C_{\tilde{\chi}_1^0\tilde{\chi}_1^0 h}$ and $C_{\tilde{\chi}_1^0\tilde{\chi}_1^0 Z}$ are independent. They can not be suppressed simultaneously for light Higgsinos!

Multi-lepton signal at the LHC:

- $S=\sum \sigma(pp\to \tilde{\chi}_i\tilde{\chi}_j, \tilde{\ell}^*\tilde{\ell})\times Br\times\epsilon$, ϵ: signal selection efficiency;
- $\sigma(pp\to \tilde{W}\tilde{W}) > \sigma(pp\to\tilde{H}\tilde{H}) > \sigma(pp\to\tilde{\ell}^*_L\tilde{\ell}_L) > \sigma(pp\to\tilde{\ell}^*_R\tilde{\ell}_R)$ for degenerated mass;
- Lepton production efficiency: $Br(\tilde{\chi}_i^0\to\tilde{\ell}^*\ell) > Br(\tilde{\chi}_j^-\to\tilde{\ell}^*\nu), Br(\tilde{\ell}\to\ell\tilde{\chi}_1^0) > Br(W\to \ell\nu), Br(Z\to\ell^*\ell) > Br(h\to\ell^*\ell)$.

MSSM

Comparing the theory with the other type DM cancidate, LHC constraints on MSSM are strong. NLSP may be a Wino- or Higgsino-dominated sparticle, or a slepton.

DM Co-annihilating with a Wino-dominated NLSP:

1. Constraints on LHC search for SUSY is relatively weak.
2. Δa_μ prefers a large μ.

DM Co-annihilating with a Higgsino-dominated NLSP:

1. DM direct detection experiments prefer an excessively large $|\mu|$.
2. Constraints on LHC search for SUSY is very weak.

DM Co-annihilating with Sletpon NLSP:

1. The Slepton may be right-handed or left-handed.
2. Constraints from LHC search for SUSY is very strong.

After considering all constraints, $|\mu| > 500$ GeV.

MSSM: more explanations

Mass insertion method: **WHL**, **WHR**, **BHR**, and **BLR** diagrams contribute to $a_\mu^{\rm SUSY}$.

$$a_{\mu,\rm WHL}^{\rm SUSY} = \frac{\alpha_2}{8\pi}\frac{m_\mu^2 \mu M_2 \tan\beta}{m_{\tilde{\nu}_\mu}^4}\left\{2 f_C\left(\frac{M_2^2}{m_{\tilde{\nu}_\mu}^4}, \frac{\mu^2}{m_{\tilde{\nu}_\mu}^2}\right) - \frac{m_{\tilde{\nu}_\mu}^4}{m_{\tilde{\mu}_L}^4} f_N\left(\frac{M_2^2}{m_{\tilde{\mu}_L}^2}, \frac{\mu^2}{m_{\tilde{\mu}_L}^2}\right)\right\}$$

$$a_{\mu,\rm BHL}^{\rm SUSY} = \frac{\alpha_Y}{8\pi}\frac{m_\mu^2 \mu M_1 \tan\beta}{m_{\tilde{\mu}_L}^4} f_N\left(\frac{M_1^2}{m_{\tilde{\mu}_L}^2}, \frac{\mu^2}{m_{\tilde{\mu}_L}^2}\right)$$

$$a_{\mu,\rm BHR}^{\rm SUSY} = -\frac{\alpha_Y}{4\pi}\frac{m_\mu^2 \mu M_1 \tan\beta}{m_{\tilde{\mu}_R}^4} f_N\left(\frac{M_1^2}{m_{\tilde{\mu}_R}^2}, \frac{\mu^2}{m_{\tilde{\mu}_R}^2}\right)$$

$$a_{\mu,\rm BLR}^{\rm SUSY} = \frac{\alpha_Y}{4\pi}\frac{m_\mu^2 \mu M_1 \tan\beta}{M_1^4} f_N\left(\frac{m_{\tilde{\mu}_L}^2}{M_1^2}, \frac{m_{\tilde{\mu}_R}^2}{M_1^2}\right)$$

Summary

Model	Annihilation process	σ^{SI}	σ^{SD}
MSSM	$\tilde{\chi}_1^0\tilde{\chi}_i^0, \tilde{\chi}_1^0\tilde{\chi}_j^\pm, \tilde{\chi}_1^0\tilde{\ell}_k, \tilde{\chi}_1^0\tilde{q}_f \to XY$, $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to Z^*, H^* \to XY$.	$\propto \left[e\tan\theta_w \frac{m_z}{\mu\left(1-m_{\tilde{\chi}_1^0}^2/\mu^2\right)} \left(\sin2\beta + m_{\tilde{\chi}_1^0}/\mu\right)\right]^2$	$\propto \left[e\tan\theta_w \frac{m_z^2\cos2\beta}{2\mu^2\left(1-m_{\tilde{\chi}_1^0}^2/\mu^2\right)}\right]^2$
Z_3-NMSSM	$\tilde{\chi}_1^0\tilde{\chi}_1^0 \to t\bar{t}$, $\tilde{\chi}_1^0\tilde{\chi}_i^0, \tilde{\chi}_1^0\tilde{\chi}_j^\pm, \to XY$.	$\propto \left[\frac{\lambda^2 v}{\mu_{\mathrm{eff}}} \frac{1}{1-m_{\tilde{\chi}_1^0}^2/\mu_{\mathrm{eff}}^2} \left(\frac{m_{\tilde{\chi}_1^0}}{\mu_{\mathrm{eff}}} - \sin2\beta\right)\right]^2$	$\propto \left[\frac{\lambda^2 v}{\mu_{\mathrm{eff}}^2} \frac{m_z\cos2\beta}{1-m_{\tilde{\chi}_1^0}^2/\mu_{\mathrm{eff}}^2}\right]^2$
GNMSSM	$\tilde{\chi}_1^0\tilde{\chi}_1^0 \to h_s A_s$	$\propto \left[\frac{\lambda^2 v}{\mu_{\mathrm{tot}}} \frac{1}{1-m_{\tilde{\chi}_1^0}^2/\mu_{\mathrm{tot}}^2} \left(\frac{m_{\tilde{\chi}_1^0}}{\mu_{\mathrm{tot}}} - \sin2\beta\right)\right]^2$	$\propto \left[\frac{\lambda^2 v}{\mu_{\mathrm{tot}}^2} \frac{m_z\cos2\beta}{1-m_{\tilde{\chi}_1^0}^2/\mu_{\mathrm{tot}}^2}\right]^2$
Type-I NMSSM	$\tilde{\nu}_1\tilde{\chi}_i^0 \to XY$, $\tilde{\nu}_1\tilde{\nu}_1 \to h_s \to XY$, $\tilde{\nu}_1\tilde{\nu}_1 \to h_s h_s, A_s A_s, \nu_h \nu_h$.	$\propto \left[-\frac{\sqrt{2}}{\lambda}\left(2\lambda_\nu^2 + \kappa\lambda_\nu\right)\mu_{\mathrm{eff}} + \frac{\lambda_\nu A_{\lambda_\nu}}{\sqrt{2}}\right]^2$	-

The status of MSSM:

Model	MSSM	Z_3-NMSSM		GNMSSM	Type-I NMSSM	B-L NMSSM
DM candidate	Bino	Bino	Singlino	Singlino	Sneutrino	Bino, Singlino, BLino, Bileptino, and sneutrino
DM physics	×	×	×	✓	✓✗	✓
LHC and Δa_μ	×	×	×	✓	✓	✓
EWSB	×	×	✓	✓	✓	✓
Neutrino	×	×	×	×	✓✗	✓
Higgs mass	×	×	×	×	✓✗	✓

Part Four

Example II: Z_3-MSSM

Z_3-NMSSM

- Field content and gauge group

SF	Spin 0	Spin $\frac{1}{2}$	Generations	$(\mathrm{U}(1) \otimes \mathrm{SU}(2) \otimes \mathrm{SU}(3))$
$\hat{q}$	$\tilde{q}$	q	3	$(\frac{1}{6}, \mathbf{2}, \mathbf{3})$
$\hat{l}$	$\tilde{l}$	l	3	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{H}_d$	H_d	$\tilde{H}_d$	1	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{H}_u$	H_u	$\tilde{H}_u$	1	$(\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{d}$	$\tilde{d}_R^*$	d_R^*	3	$(\frac{1}{3}, \mathbf{1}, \overline{\mathbf{3}})$
$\hat{u}$	$\tilde{u}_R^*$	u_R^*	3	$(-\frac{2}{3}, \mathbf{1}, \overline{\mathbf{3}})$
$\hat{e}$	$\tilde{e}_R^*$	e_R^*	3	$(1, \mathbf{1}, \mathbf{1})$
$\hat{s}$	S	$\tilde{S}$	1	$(0, \mathbf{1}, \mathbf{1})$

- Superpotential: **Try to solve μ- and little hierarchy problems of the MSSM. There is no dimensional parameters in the superpotential. However, once Z_3-symmetry was spontaneously broken, domain wall problem and tadpole problem will be induced! Occam's razor was used incorrectly.**

$$W_{\mathrm{NMSSM}} = W_{\mathrm{Yukawa}} + \lambda \hat{S} \hat{H}_u \hat{H}_d + \frac{1}{3} \kappa \hat{S}^3$$

Z_3-NMSSM

- Neutralino mass matrix

$$\mathcal{M} = \begin{pmatrix} M_1 & 0 & -\frac{g_1 v_d}{\sqrt{2}} & \frac{g_1 v_u}{\sqrt{2}} & 0 \\ & M_2 & \frac{g_2 v_d}{\sqrt{2}} & -\frac{g_2 v_u}{\sqrt{2}} & 0 \\ & & 0 & -\mu_{eff} & -\lambda v_u \\ & & & 0 & -\lambda v_d \\ & & & & \frac{2\kappa}{\lambda}\mu_{eff} \end{pmatrix}$$

- DM may be Singlino-dominated. Its mass and couplings are given by

$$m_{\tilde{\chi}_1^0} \simeq \frac{2\kappa}{\lambda}\mu + \frac{\lambda^2 v^2}{\mu^2}(\mu \sin 2\beta - \frac{2\kappa}{\lambda}\mu), \quad C_{\tilde{\chi}_1^0\tilde{\chi}_1^0 Z} \simeq \frac{m_Z}{\sqrt{2}v}(\frac{\lambda v}{\mu_{eff}})^2 \frac{\cos 2\beta}{1-(m_{\tilde{\chi}_1^0}/\mu_{eff})^2} \quad ,$$

$$C_{\tilde{\chi}_1^0\tilde{\chi}_1^0 h} \simeq \sqrt{2}\lambda \left(\frac{\lambda v}{\mu_{eff}}\right) \frac{V_{h_i}^{\rm SM}(m_{\tilde{\chi}_1^0}/\mu_{eff} - \sin 2\beta)}{1-(m_{\tilde{\chi}_1^0}/\mu_{eff})^2} + \ldots$$

- DM properties are described by four independent parameters: λ, μ_{eff}, $m_{\tilde{\chi}_1^0}$, and $\tan\beta$.

- $m_{\tilde{\chi}_1^0}$ and κ are correlated, λ and κ are correlated by $2\kappa \,/\, \lambda < 1$;
- Small λ is preferred to suppress DM-nucleon scattering.

Z_3-NMSSM: Dominant annihilation channels

Conditions to obtain the measured DM abundance:

1. $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to t\bar{t}$: s-channel exchange of Z and Higgs bosons.

$$|C_{\tilde{\chi}_1^0\tilde{\chi}_1^0 G^0}| = \frac{\sqrt{2}m_{\tilde{\chi}_1^0}}{v}\left(\frac{\lambda v}{\mu_{eff}}\right)^2 \frac{\cos 2\beta}{1-(m_{\tilde{\chi}_1^0}/\mu_{eff})^2} \simeq 0.1.$$

2. $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to h_s A_s$: s-channel exchange of Higgs bosons, t-channel exchange of neutralinos.

$$|C_{\tilde{\chi}_1^0\tilde{\chi}_1^0 h_s}| = |C_{\tilde{\chi}_1^0\tilde{\chi}_1^0 A_s}| = -\sqrt{2}\kappa \simeq 0.2 \times \left(\frac{m_{\tilde{\chi}_1^0}}{300\ \text{GeV}}\right)^{1/2}.$$

3. $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to h A_s$: s-channel exchange of Higgs bosons, t-channel exchange of neutralinos.

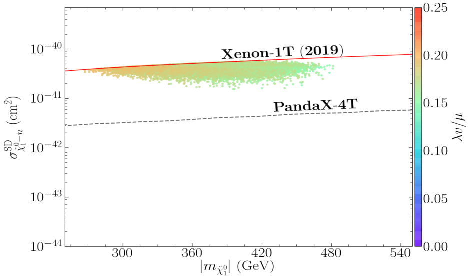
$$\lambda^3 \sin 2\beta \simeq \left(\frac{\mu_{eff}}{700\ \text{GeV}}\right)^2.$$

$\lambda > 0.3$ is preferred to predict the measured abundance.

Z_3-NMSSM

Favored parameter space

1. Type-I samples (h_1 as SM-like Higgs boson) : $0.4 \lesssim \lambda \lesssim 0.7$, $0.13 \lesssim \kappa \lesssim 0.23$, $1.5 \lesssim \tan\beta \lesssim 6$, 450 GeV $\lesssim \mu_{eff} \lesssim 720$ GeV, and $\ln Z_1 = -24.2$;
2. Type-II samples (h_1 as SM-like Higgs boson):$\lambda \lesssim 0.08$, $-0.04 \lesssim \kappa < 0$, $4 \lesssim \tan\beta \lesssim 24$, 170 GeV $\lesssim \mu_{eff} \lesssim 420$ GeV, and $\ln Z_2 = -27.5$;
3. Type-III samples (h_2 as SM-like Higgs boson) : $\lambda \lesssim 0.15$, $|\kappa| \lesssim 0.06$, $4.5 \lesssim \tan\beta \lesssim 32$, 135 GeV $\lesssim \mu_{eff} \lesssim 260$ GeV, and $\ln Z_3 = -27.0$.



Since $\tan\beta$ is moderately small, light sparticles are preferred to explain the Δa_μ anomaly. This situation is limited very tightly by the LHC search for SUSY.

Summary

Model	Annihilation process	$\sigma^{\rm SI}$	$\sigma^{\rm SD}$
MSSM	$\tilde\chi_1^0\tilde\chi_i^0, \tilde\chi_1^0\tilde\chi_j^\pm, \tilde\chi_1^0\tilde\ell_k, \tilde\chi_1^0\tilde q_f \to XY,$ $\tilde\chi_1^0\tilde\chi_1^0 \to Z^*, H^* \to XY.$	$\propto \left[e\tan\theta_w \frac{m_z}{\mu\left(1-m^2_{\tilde\chi_1^0}/\mu^2\right)} \left(\sin2\beta + m_{\tilde\chi_1^0}/\mu\right)\right]^2$	$\propto \left[e\tan\theta_w \frac{m_z^2\cos2\beta}{2\mu^2\left(1-m^2_{\tilde\chi_1^0}/\mu^2\right)}\right]^2$
Z_3-NMSSM	$\tilde\chi_1^0\tilde\chi_1^0 \to t\bar t,$ $\tilde\chi_1^0\tilde\chi_i^0, \tilde\chi_1^0\tilde\chi_j^\pm, \to XY.$	$\propto \left[\frac{\lambda^2 v}{\mu_{\rm eff}} \frac{1}{1-m^2_{\tilde\chi_1^0}/\mu^2_{\rm eff}} \left(\frac{m_{\tilde\chi_1^0}}{\mu_{\rm eff}} - \sin2\beta\right)\right]^2$	$\propto \left[\frac{\lambda^2 v}{\mu^2_{\rm eff}} \frac{m_z\cos2\beta}{1-m^2_{\tilde\chi_1^0}/\mu^2_{\rm eff}}\right]^2$
GNMSSM	$\tilde\chi_1^0\tilde\chi_1^0 \to h_s A_s$	$\propto \left[\frac{\lambda^2 v}{\mu_{\rm tot}} \frac{1}{1-m^2_{\tilde\chi_1^0}/\mu^2_{\rm tot}} \left(\frac{m_{\tilde\chi_1^0}}{\mu_{\rm tot}} - \sin2\beta\right)\right]^2$	$\propto \left[\frac{\lambda^2 v}{\mu^2_{\rm tot}} \frac{m_z\cos2\beta}{1-m^2_{\tilde\chi_1^0}/\mu^2_{\rm tot}}\right]^2$
Type-I NMSSM	$\tilde\nu_1\tilde\chi_i^0 \to XY,\ \tilde\nu_1\tilde\nu_1 \to h_s \to XY,$ $\tilde\nu_1\tilde\nu_1 \to h_s h_s, A_s A_s, \nu_h\nu_h.$	$\propto \left[-\frac{\sqrt2}{\lambda}\left(2\lambda_\nu^2 + \kappa\lambda_\nu\right)\mu_{\rm eff} + \frac{\lambda_\nu A_{\lambda_\nu}}{\sqrt2}\right]^2$	-

The status of Z_3-NMSSM:

Model	MSSM	Z_3-NMSSM		GNMSSM	Type-I NMSSM	B-L NMSSM
DM candidate	Bino	Bino	Singlino	Singlino	Sneutrino	Bino, Singlino, BLino, Bileptino, and sneutrino
DM physics	×	×	×	✓	✓✗	✓
LHC and Δa_μ	×	×	×	✓	✓	✓
EWSB	×	×	✓	✓	✓	✓
Neutrino	×	×	×	×	✓✗	✓
Higgs mass	×	×	×	×	✓✗	✓

The singlino-dominated DM scenario in Z_3-NMSSM has been tightly limited. The phenomenology of the bino-dominated DM scenario is roughly same as that of the MSSM.

Part Four

Example III: General MSSM

GNMSSM: Motivation and superpotential

- Chiral Superfields

SF	Spin 0	Spin $\frac{1}{2}$	Generations	$(\mathrm{U}(1) \otimes \mathrm{SU}(2) \otimes \mathrm{SU}(3))$
$\hat{q}$	$\tilde{q}$	q	3	$(\frac{1}{6}, \mathbf{2}, \mathbf{3})$
$\hat{l}$	$\tilde{l}$	l	3	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{H}_d$	H_d	$\tilde{H}_d$	1	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{H}_u$	H_u	$\tilde{H}_u$	1	$(\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{d}$	$\tilde{d}_R^*$	d_R^*	3	$(\frac{1}{3}, \mathbf{1}, \overline{\mathbf{3}})$
$\hat{u}$	$\tilde{u}_R^*$	u_R^*	3	$(-\frac{2}{3}, \mathbf{1}, \overline{\mathbf{3}})$
$\hat{e}$	$\tilde{e}_R^*$	e_R^*	3	$(1, \mathbf{1}, \mathbf{1})$
$\hat{s}$	S	$\tilde{S}$	1	$(0, \mathbf{1}, \mathbf{1})$

- Superpotential

$$W_{\mathrm{GNMSSM}} = W_{\mathrm{Yukawa}} + \lambda \hat{S} \hat{H}_u \cdot \hat{H}_d + \frac{1}{3}\kappa \hat{S}^3 + \mu \hat{H}_u \cdot \hat{H}_d + \frac{1}{2}\nu \hat{S}^2 + \xi \hat{S}$$

1. Solve domain wall and tadpole problems in Z_3-NMSSM.
2. Z_3-violating terms from an underlying theory with Z_4^n or Z_8^n symmetry.

GNMSSM: DM mass and couplings

- Neutralino mass matrix

$$m_{\tilde{\chi}_i^0} = \begin{pmatrix} M_1 & 0 & -\frac{1}{2}g_1 v_d & \frac{1}{2}g_1 v_u & 0 \\ 0 & M_2 & \frac{1}{2}g_2 v_d & -\frac{1}{2}g_2 v_u & 0 \\ -\frac{1}{2}g_1 v_d & \frac{1}{2}g_2 v_d & 0 & -\mu - \mu_{\rm eff} & -\frac{1}{\sqrt{2}}v_u\lambda \\ \frac{1}{2}g_1 v_u & -\frac{1}{2}g_2 v_u & -\mu - \mu_{\rm eff} & 0 & -\frac{1}{\sqrt{2}}v_d\lambda \\ 0 & 0 & -\frac{1}{\sqrt{2}}v_u\lambda & -\frac{1}{\sqrt{2}}v_d\lambda & \frac{2\kappa}{\lambda}\mu_{\rm eff} \end{pmatrix}$$

Couplings of the singlino-dominated DM are given by:

$$C_{\tilde{\chi}_1^0\tilde{\chi}_1^0 h} \simeq \lambda \frac{\lambda v}{\mu + \mu_{\rm eff}} \frac{V_{h_i}^{\rm SM}(m_{\tilde{\chi}_1^0}/(\mu + \mu_{\rm eff}) - \sin 2\beta)}{1 - (m_{\tilde{\chi}_1^0}/(\mu + \mu_{\rm eff}))^2} + \ldots$$

$$C_{\tilde{\chi}_1^0\tilde{\chi}_1^0 Z} \simeq \frac{m_Z}{2v}\left(\frac{\lambda v}{\mu + \mu_{\rm eff}}\right)^2 \frac{\cos 2\beta}{1 - (m_{\tilde{\chi}_1^0}/(\mu + \mu_{\rm eff}))^2} ,$$

- **DM mass and κ are not correlated, λ and κ are not correlated!**
- DM properties are described by **five** independent parameters: $m_{\tilde{\chi}_1^0}$, λ, κ, $\tan\beta$, and $\mu_{\rm tot} \equiv \mu + \mu_{\rm eff}$.
- Small λ is preferred to suppress DM-nucleon scatterings.

GNMSSM: Dominant annihilation channels

Conditions to obtain the measured DM abundance:

1. $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to t\bar{t}$: s-channel exchange of Z and Higgs bosons.

$$|C_{\tilde{\chi}_1^0\tilde{\chi}_1^0 G^0}| = \frac{\sqrt{2} m_{\tilde{\chi}_1^0}}{v} \left(\frac{\lambda v}{\mu_{tot}}\right)^2 \frac{\cos 2\beta}{1 - (m_{\tilde{\chi}_1^0}/\mu_{tot})^2} \simeq 0.1.$$

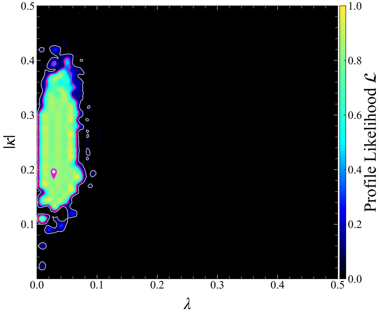
2. $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to h_s A_s$: s-channel exchange of Higgs bosons, t-channel exchange of neutralinos.

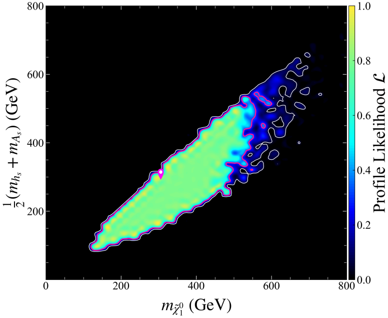
$$|C_{\tilde{\chi}_1^0\tilde{\chi}_1^0 h_s}| = |C_{\tilde{\chi}_1^0\tilde{\chi}_1^0 A_s}| = -\sqrt{2}\kappa \simeq 0.2 \times \left(\frac{m_{\tilde{\chi}_1^0}}{300\ \text{GeV}}\right)^{1/2}.$$

3. $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to h A_s$: s-channel exchange of Higgs bosons, t-channel exchange of neutralinos.

$$\lambda^3 \sin 2\beta \simeq \left(\frac{\mu}{700\ \text{GeV}}\right)^2.$$

Singlet-dominated particles may form a secluded DM sector:
measured abundance obtained by $\tilde{\chi}_1^0\tilde{\chi}_1^0 \to h_s A_s$ (via adjusting κ);
DM-nucleon scatterings suppressed by a small $\lambda v/\mu_{tot}$.
The simplest SUSY framework to realize secluded DM sector.





Parameter space:

$$0 < \lambda \leq 0.70, \quad |\kappa| \leq 0.70, \quad 1 \leq \tan\beta \leq 60, \quad |A_t| \leq 5 \text{ TeV},$$
$$0 < \mu_{eff} \leq 500 \text{ GeV}, \quad 100 \text{ GeV} \leq |\mu_{tot}| \leq 500 \text{ GeV}, \quad |A_\kappa| \leq 1000 \text{ GeV}.$$

Likelihood function:

$$\mathcal{L} = \mathcal{L}_{\Omega h^2} \times \mathcal{L}_{DD} \times \mathcal{L}_{IDD} \times \mathcal{L}_{Higgs} \times \mathcal{L}_B$$

GNMSSM: Dominant annihilation channels

$h \equiv h_1$ scenario: lnZ = -65.79 $\pm$ 0.046			
$\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to \mathrm{h_s\ A_s}$	$\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to \mathrm{t\ \bar{t}}$	$\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to \mathrm{h_s\ h_s}$	Co-annihilation
88%	8%	3%	0.7%
$h \equiv h_2$ scenario: lnZ = -68.23 $\pm$ 0.051			
$\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to \mathrm{h_s\ A_s}$	$\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to \mathrm{t\ \bar{t}}$	Co-annihilation	h-funnel
76%	12%	11.6%	0.3%

Table 1: Dominant annihilation channels and their normalized posterior probabilities for $h \equiv h_1$ and $h \equiv h_2$ scenarios. In obtaining the values in this table, each sample's most critical channel for the abundance was identified and sequentially used to classify the samples. The posterior probability densities of the same type of samples were then summed.

$\tilde{\chi}_1^0 \tilde{\chi}_1^0 \to h_s A_s$ **always played a role in DM annihilation.**

GNMSSM: Explaining the Muon g-2 anomaly

Characteristics:

1. Roughly same loop contributions as the MSSM.
2. **DM physics is changed**.
3. **LHC constraints is alleviated significantly**.
4. **Vacuum becomes more stable**.

Mechanism to alleviate the LHC constraints:

1. DM must be heavy to achieve the measured relic density.
2. For singlino-dominated DM, heavy sparticles prefer to decay into NLSP or NNLSP first; their decay chains are lengthened.
3. Light singlet Higgs bosons may act as the sparticle decay products.

GNMSSM: Explaining the Muon g-2 anomaly

- The following processes are considered:

$$
\begin{aligned}
pp &\to \tilde{\chi}_i^0 \tilde{\chi}_j^{\pm}, & i = 2,3,4,5; \quad j = 1,2 \\
pp &\to \tilde{\chi}_i^{\pm} \tilde{\chi}_j^{\mp}, & i = 1,2; \quad j = 1,2 \\
pp &\to \tilde{\chi}_i^0 \tilde{\chi}_j^0, & i = 2,3,4,5; \quad j = 2,3,4,5 \\
pp &\to \tilde{\mu}_i \tilde{\mu}_j, & i = L,R; \quad j = L,R
\end{aligned}
$$

- All LHC searches for electroweakinos and sleptons are considered, a total of 14 analyses for Run-II data.
- Newly added analyses:
  1. ATLAS search for 3 lepton plus missing E_T signal, see CERN-EP-2021-059, or arXiv: 2106.01676.
  2. CMS search for 2 lepton plus missing E_T signal, arXiv: 2012.08600.

GNMSSM: Explaining the Muon g-2 anomaly

Analysis	Simplified Scenario	Signal of Final State	Luminosity
CMS-SUS-17-010 (arXiv:1807.07799)	$\tilde{\chi}_1^{\pm}\tilde{\chi}_1^{\mp} \to W^{\pm}\tilde{\chi}_1^0 W^{\mp}\tilde{\chi}_1^0$ $\tilde{\chi}_1^{\pm}\tilde{\chi}_1^{\mp} \to \nu\tilde{\ell}/\ell\tilde{\nu} \to \ell\ell\nu\nu\tilde{\chi}_1^0\tilde{\chi}_1^0$	$2\ell + E_{\rm T}^{\rm miss}$	35.9 fb^{-1}
CMS-SUS-17-009 (arXiv:1806.05264)	$\tilde{\ell}\tilde{\ell} \to \ell\ell\tilde{\chi}_1^0\tilde{\chi}_1^0$	$2\ell + E_{\rm T}^{\rm miss}$	35.9 fb^{-1}
CMS-SUS-17-004 (arXiv:1801.03957)	$\tilde{\chi}_2^0\tilde{\chi}_1^{\pm} \to Wh(Z)\tilde{\chi}_1^0\tilde{\chi}_1^0$	$\mathrm{n}\ell(\geq 0) + \mathrm{n}j(\geq 0) + E_{\rm T}^{\rm miss}$	35.9 fb^{-1}
CMS-SUS-16-045 (arXiv:1709.00384)	$\tilde{\chi}_2^0\tilde{\chi}_1^{\pm} \to W^{\pm}\tilde{\chi}_1^0 h\tilde{\chi}_1^0$	$1\ell 2b + E_{\rm T}^{\rm miss}$	35.9 fb^{-1}
CMS-SUS-16-039 (arxiv:1709.05406)	$\tilde{\chi}_2^0\tilde{\chi}_1^{\pm} \to \ell\tilde{\nu}\ell\tilde{\ell}$ $\tilde{\chi}_2^0\tilde{\chi}_1^{\pm} \to \tilde{\tau}\nu\tilde{\ell}\ell$ $\tilde{\chi}_2^0\tilde{\chi}_1^{\pm} \to \tilde{\tau}\nu\tilde{\tau}\tau$ $\tilde{\chi}_2^0\tilde{\chi}_1^{\pm} \to WZ\tilde{\chi}_1^0\tilde{\chi}_1^0$ $\tilde{\chi}_2^0\tilde{\chi}_1^{\pm} \to WH\tilde{\chi}_1^0\tilde{\chi}_1^0$	$n\ell(\geq 0)(\tau) + E_{\rm T}^{\rm miss}$	35.9 fb^{-1}
CMS-SUS-16-034 (arXiv:1709.08908)	$\tilde{\chi}_2^0\tilde{\chi}_1^{\pm} \to W\tilde{\chi}_1^0 Z(h)\tilde{\chi}_1^0$	$\mathrm{n}\ell(\geq 2) + \mathrm{n}j(\geq 1)E_{\rm T}^{\rm miss}$	35.9 fb^{-1}
CERN-EP-2017-303 (arXiv:1803.02762)	$\tilde{\chi}_2^0\tilde{\chi}_1^{\pm} \to WZ\tilde{\chi}_1^0\tilde{\chi}_1^0$ $\tilde{\chi}_2^0\tilde{\chi}_1^{\pm} \to \nu\tilde{\ell}\ell\tilde{\ell}$ $\tilde{\chi}_1^{\pm}\tilde{\chi}_1^{\mp} \to \nu\tilde{\ell}/\ell\tilde{\nu} \to \ell\ell\nu\nu\tilde{\chi}_1^0\tilde{\chi}_1^0$ $\tilde{\ell}\tilde{\ell} \to \ell\ell\tilde{\chi}_1^0\tilde{\chi}_1^0$	$\mathrm{n}\ell(\geq 2) + E_{\rm T}^{\rm miss}$	35.9 fb^{-1}
CERN-EP-2018-306 (arXiv:1812.09432)	$\tilde{\chi}_2^0\tilde{\chi}_1^{\pm} \to Wh\tilde{\chi}_1^0\tilde{\chi}_1^0$	$\mathrm{n}\ell(\geq 0) + \mathrm{n}j(\geq 0) + \mathrm{n}b(\geq 0) + \mathrm{n}\gamma(\geq 0) + E_{\rm T}^{\rm miss}$	35.9 fb^{-1}
CERN-EP-2018-113 (arXiv:1806.02293)	$\tilde{\chi}_2^0\tilde{\chi}_1^{\pm} \to WZ\tilde{\chi}_1^0\tilde{\chi}_1^0$	$\mathrm{n}\ell(\geq 2) + \mathrm{n}j(\geq 0) + E_{\rm T}^{\rm miss}$	35.9 fb^{-1}
CERN-EP-2019-263 (arXiv:1912.08479)	$\tilde{\chi}_2^0 v\tilde{\chi}_1^{\pm} \to W\tilde{\chi}_1^0 Z\tilde{\chi}_1^0 \to \ell\nu\ell\ell\tilde{\chi}_1^0\tilde{\chi}_1^0$	$3\ell + E_{\rm T}^{\rm miss}$	139 fb^{-1}
CERN-EP-2019-106 (arXiv:1908.08215)	$\tilde{\ell}\tilde{\ell} \to \ell\ell\tilde{\chi}_1^0\tilde{\chi}_1^0$ $\tilde{\chi}_1^{\pm}\tilde{\chi}_1^{\mp} \to \nu\tilde{\ell}/\ell\tilde{\nu} \to \ell\ell\nu\nu\tilde{\chi}_1^0\tilde{\chi}_1^0$	$2\ell + E_{\rm T}^{\rm miss}$	139 fb^{-1}
CERN-EP-2019-188 (arXiv:1909.09226)	$\tilde{\chi}_2^0\tilde{\chi}_1^{\pm} \to Wh\tilde{\chi}_1^0\tilde{\chi}_1^0$	$1\ell + h(\to bb) + E_{\rm T}^{\rm miss}$	139 fb^{-1}

Table 2: Signal of final state for electroweakino pair-production processes.

GNMSSM: Explaining the Muon g-2 anomaly

Consider h_1 as the SM-Like Higgs boson.

- Constraints on different NLSP.

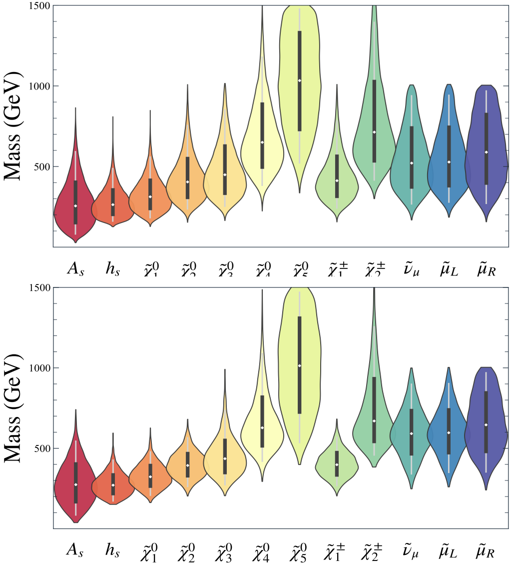
NLSP	$m_{\tilde{\chi}_1^0}$	$\mu + \mu_{\rm eff}$	M_2	$m_{\tilde{\mu}_L}$	$m_{\tilde{\mu}_R}$	$N_{\rm tot}$	$N_{\rm pass}^{\rm MC}$	$N_{\rm pass}^{\rm VS}$
$\tilde{\nu}_\mu$	200	250	370	250	300	1751	**127**	124
$\tilde{\mu}_R$	200	300	350	350	300	1071	**24**	24
$\tilde{B}$	200	300	300	350	350	310	**103**	103
$\tilde{W}$	200	300	250	350	350	1246	**792**	784
$\tilde{H}$	160	200	300	250	250	3162	**1606**	1606

Table 3: Summarization of the samples classified by their NLSP's dominant component. $N_{\rm tot}$ represents the total number of each type of samples surveyed by concrete Monte Carlo simulations. $N_{\rm pass}^{\rm MC}$ represents the corresponding number satisfying $R < 1$, and $N_{\rm pass}^{\rm VS}$ are that further stasfying vacuum stability constraint. The lower limits of parameters $(\mu + \mu_{\rm eff})$, M_2, $m_{\tilde{\chi}_1^0}$, $m_{\tilde{\mu}_L}$ and $m_{\tilde{\mu}_R}$ for the samples surviving the constraints are given in units of GeV in each row.

About two thirds samples have been excluded!

GNMSSM: Explaining the Muon g-2 anomaly

- Mass spectra before and after considering the LHC constraints.



GNMSSM: Explaining the Muon g-2 anomaly

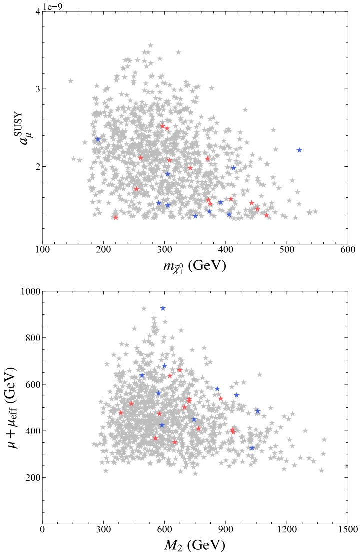


Figure 4: The samples with $\tilde{\mu}_R$-dominated NLSP.

GNMSSM: Explaining the Muon g-2 anomaly

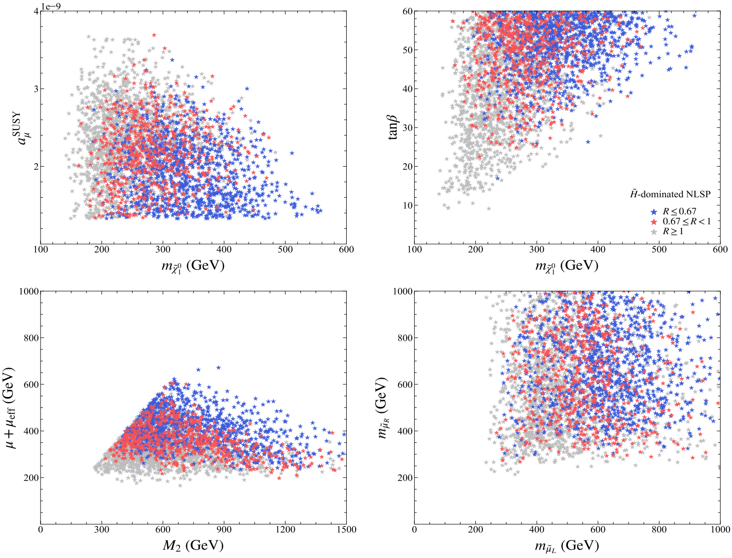
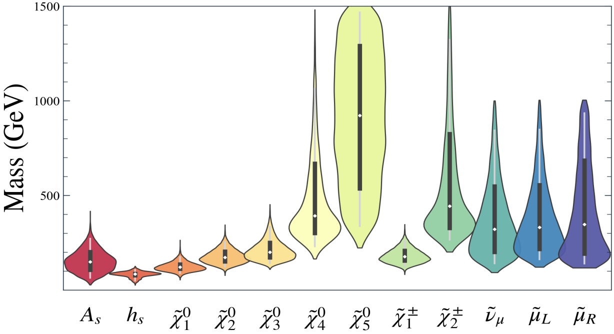


Figure 5: The samples with $\tilde{H}$-dominated NLSP.

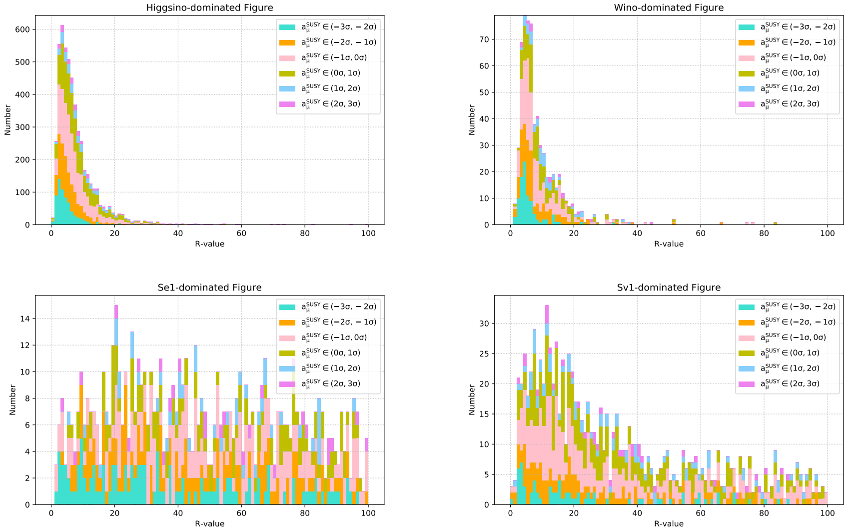
Results for h_2 as the SM-Like Higgs boson.

- Mass spectra for the samples of h_2 scenario before considering the LHC constraints.



GNMSSM: Explaining the Muon g-2 anomaly

- R-value for the samples in the h_2 scenario. **Strong tension**!



Summary

Model	Annihilation process	$\sigma^{\rm SI}$	$\sigma^{\rm SD}$
MSSM	$\tilde\chi_1^0\tilde\chi_i^0, \tilde\chi_1^0\tilde\chi_j^\pm, \tilde\chi_1^0\tilde\ell_k, \tilde\chi_1^0\tilde q_f \to XY$, $\tilde\chi_1^0\tilde\chi_1^0 \to Z^*, H^* \to XY$.	$\propto \left[e\tan\theta_w \frac{m_z}{\mu\left(1-m^2_{\tilde\chi_1^0}/\mu^2\right)} \left(\sin2\beta + m_{\tilde\chi_1^0}/\mu\right)\right]^2$	$\propto \left[e\tan\theta_w \frac{m_z^2\cos2\beta}{2\mu^2\left(1-m^2_{\tilde\chi_1^0}/\mu^2\right)}\right]^2$
Z_3-NMSSM	$\tilde\chi_1^0\tilde\chi_1^0 \to t\bar t$, $\tilde\chi_1^0\tilde\chi_i^0, \tilde\chi_1^0\tilde\chi_j^\pm, \to XY$.	$\propto \left[\frac{\lambda^2 v}{\mu_{\rm eff}} \frac{1}{1-m^2_{\tilde\chi_1^0}/\mu^2_{\rm eff}} \left(\frac{m_{\tilde\chi_1^0}}{\mu_{\rm eff}} - \sin2\beta\right)\right]^2$	$\propto \left[\frac{\lambda^2 v}{\mu^2_{\rm eff}} \frac{m_z\cos2\beta}{1-m^2_{\tilde\chi_1^0}/\mu^2_{\rm eff}}\right]^2$
GNMSSM	$\tilde\chi_1^0\tilde\chi_1^0 \to h_s A_s$	$\propto \left[\frac{\lambda^2 v}{\mu_{\rm tot}} \frac{1}{1-m^2_{\tilde\chi_1^0}/\mu^2_{\rm tot}} \left(\frac{m_{\tilde\chi_1^0}}{\mu_{\rm tot}} - \sin2\beta\right)\right]^2$	$\propto \left[\frac{\lambda^2 v}{\mu^2_{\rm tot}} \frac{m_z\cos2\beta}{1-m^2_{\tilde\chi_1^0}/\mu^2_{\rm tot}}\right]^2$
Type-I NMSSM	$\tilde\nu_1\tilde\chi_i^0 \to XY$, $\tilde\nu_1\tilde\nu_1 \to h_s \to XY$, $\tilde\nu_1\tilde\nu_1 \to h_s h_s, A_s A_s, \nu_h\nu_h$.	$\propto \left[-\frac{\sqrt2}{\lambda}\left(2\lambda_\nu^2 + \kappa\lambda_\nu\right)\mu_{\rm eff} + \frac{\lambda_\nu A_{\lambda_\nu}}{\sqrt2}\right]^2$	-

GNMSSM: Current Status

Model	MSSM	Z_3-NMSSM		GNMSSM	Type-I NMSSM	B-L NMSSM
DM candidate	Bino	Bino	Singlino	Singlino	Sneutrino	Bino, Singlino, BLino, Bileptino, and sneutrino
DM physics	×	×	×	✓	✓✗	✓
LHC and Δa_μ	×	×	×	✓	✓	✓
EWSB	×	×	✓	✓	✓	✓
Neutrino	×	×	×	×	✓✗	✓
Higgs mass	×	×	×	×	✓✗	✓

Part Five

Example IV: Type-I MSSM

Type-I NMSSM

- Chiral Superfields

SF	Spin 0	Spin $\frac{1}{2}$	Generations	(U(1) ⊗ SU(2) ⊗ SU(3))
$\hat{q}$	$\tilde{q}$	q	3	$(\frac{1}{6}, \mathbf{2}, \mathbf{3})$
$\hat{l}$	$\tilde{l}$	l	3	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{H}_d$	H_d	$\tilde{H}_d$	1	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{H}_u$	H_u	$\tilde{H}_u$	1	$(\frac{1}{2}, \mathbf{2}, \mathbf{1})$
$\hat{d}$	$\tilde{d}_R^*$	d_R^*	3	$(\frac{1}{3}, \mathbf{1}, \overline{\mathbf{3}})$
$\hat{u}$	$\tilde{u}_R^*$	u_R^*	3	$(-\frac{2}{3}, \mathbf{1}, \overline{\mathbf{3}})$
$\hat{e}$	$\tilde{e}_R^*$	e_R^*	3	$(1, \mathbf{1}, \mathbf{1})$
$\hat{\nu}$	$\tilde{\nu}_R^*$	ν_R^*	3	$(0, \mathbf{1}, \mathbf{1})$
$\hat{s}$	S	$\tilde{S}$	1	$(0, \mathbf{1}, \mathbf{1})$

- Superpotential

$$W_{\mathrm{Type-I}} = W_{\mathrm{Yukawa}} + \lambda \hat{S}\hat{H}_u \cdot \hat{H}_d + \frac{1}{3}\kappa \hat{S}^3 + \bar{\lambda}_\nu \hat{s}\hat{\nu}\hat{\nu} + Y_\nu \hat{l} \cdot \hat{H}_u \hat{\nu}$$

1. Provide mechanisms to generate neutrino mass and mixing, and leptogenesis.
2. Lightest sneutrino may act as a feasible DM candidate.

Type-I NMSSM

- Sneutrino mass matrix

$$\mathcal{M}^2_{\tilde{\nu}} = \begin{pmatrix} m^2_{L\bar{L}} & \frac{m^2_{LR}+m^2_{L\bar{R}}+\text{c.c}}{2} & 0 & i\frac{m^2_{LR}-m^2_{L\bar{R}}-\text{c.c}}{2} \\ \frac{m^2_{LR}+m^2_{L\bar{R}}+\text{c.c}}{2} & m^2_{R\bar{R}}+m^2_{RR}+m^{2*}_{RR} & i\frac{m^2_{LR}-m^2_{L\bar{R}}-\text{c.c}}{2} & i\left(m^2_{RR}-m^{2*}_{RR}\right) \\ 0 & i\frac{m^2_{LR}-m^2_{L\bar{R}}-\text{c.c}}{2} & m^2_{L\bar{L}} & \frac{-m^2_{LR}+m^2_{L\bar{R}}+\text{c.c}}{2} \\ i\frac{m^2_{LR}-m^2_{L\bar{R}}-\text{ c.c}}{2} & i\left(m^2_{RR}-m^{2*}_{RR}\right) & \frac{-m^2_{LR}+m^2_{L\bar{R}}+\text{c.c}}{2} & m^2_{R\bar{R}}-m^2_{RR}-m^{2*}_{RR} \end{pmatrix}$$

1. Chiral mixing can be neglected. DM may be purely right-handed sneutrino.
2. Lepton number violating interactions split right-handed CP-even and CP-odd snuetrinos.

- Expression of DM-nucleon scattering cross section:

$$\sigma^{\rm SI}_{\tilde{\nu}_1-N} \simeq 4.2\times 10^{-44}\ {\rm cm}^2 \times \left(\frac{125{\rm GeV}}{m_h}\right)^4 \times \left(\frac{C_{\tilde{\nu}_1^*\tilde{\nu}_1\ {\rm Re}[S]}}{m_{\tilde{\nu}_1}} \times \delta \sin\theta\cos\theta - \frac{\cos\beta C_{\tilde{\nu}_1^*\tilde{\nu}_1\ {\rm Re}[H_d^0]} + \sin\beta C_{\tilde{\nu}_1^*\tilde{\nu}_1\ {\rm Re}[H_u^0]}}{m_{\tilde{\nu}_1}} \times \left(1+\delta\sin^2\theta\right)\right)^2$$

where

$$C_{\tilde{\nu}_1\tilde{\nu}_1 h_i} = \frac{\lambda\lambda_\nu M_W}{g}\left(\sin\beta Z_{i1} + \cos\beta Z_{i2}\right) - \left[\frac{\sqrt{2}}{\lambda}\left(2\lambda_\nu^2 + \kappa\lambda_\nu\right)\mu - \frac{\lambda_\nu A_{\lambda_\nu}}{\sqrt{2}}\right] Z_{i3},$$

$$\delta = m_h^2/m_{h_s}^2 - 1.$$

DM-nucleon scatterings are naturally suppressed!
A general conclusion for singlet-dominated DM.

Type-I NMSSM

DM annihilation channels

- $\tilde{\nu}_1 \tilde{H} \to XY, \tilde{H}\tilde{H}' \to X'Y'$:
  $m_{\tilde{\nu}_1} \simeq \mu$, co-annihilate with Higgsino-dominated electroweakinos.
- $\tilde{\nu}_1 \tilde{\nu}_1 \to SS^*$:
  s-channel Higgs exchange, t/uchannel sneutrino exchange, and a four-point interaction.
- $\tilde{\nu}_1 \tilde{\nu}_1 \to \nu_R \bar{\nu}_R$:
  s-channel Higgs exchange and t/u-channel neutralino exchange.
- $\tilde{\nu}_1 \tilde{\nu}_1 \to VV^*, VS, f\bar{f}$:
  s-channel Higgs exchange.

In the DM annihilation processes, the singlet field as a propagator and final states contributing the most to the correct residual density of dark matter.

Type-I NMSSM

CP-even light h_s scenario: $\ln Z = -40.7 \pm 0.20$			
Annihilation characteristics		Percent	
Coannihilation	$\tilde{\nu}_1 \chi_1 \to XY$	37%	38.2%
	$\tilde{\nu}_1 \tilde{\nu}_1^I \to \nu_4 \nu_4$	1.1%	
	$\tilde{\nu}_1^I \tilde{\nu}_1^I \to h_s h_s$	0.1%	
Secluded DM sector	$\tilde{\nu}_1 \tilde{\nu}_1 \to A_s A_s$	0.2%	55.3%
	$\tilde{\nu}_1 \tilde{\nu}_1 \to h_s h_s$	54%	
	$\tilde{\nu}_1 \tilde{\nu}_1 \to \nu_4 \nu_4$	1.1%	
Higgs portal	$\tilde{\nu}_1 \tilde{\nu}_1 \to h h_s$	0.2%	6.5%
	$\tilde{\nu}_1 \tilde{\nu}_1 \to hh$	0.3%	
	$\tilde{\nu}_1 \tilde{\nu}_1 \to gg$	0.1%	
	$\tilde{\nu}_1 \tilde{\nu}_1 \to b\bar{b}$	1.8%	
	$\tilde{\nu}_1 \tilde{\nu}_1 \to W^+ W^-$	4.1%	

Table 4: The annihilation mechanisms and channels in CP-even light h_s scenario, where $\chi_1 \equiv \{\tilde{\chi}_1^\pm, \tilde{\chi}_1^0, \tilde{\chi}_2^0\}$, X and Y represent any possible final states, and $\tilde{\nu}_1^I$ denotes the lightest CP-odd sneutrino particle.

Type-I NMSSM

CP-even heavy h_s scenario: $\ln Z = -31.8 \pm 0.02$			
Annihilation characteristics		Percent	
Coannihilation	$\tilde{\nu}_1 \chi_2 \to \mathrm{XY}$	85%	86.1%
	$\tilde{\nu}_1 \tilde{\nu}_1^{\mathrm{I}} \to \mathrm{Y_1 Y_2}$	1.0%	
	$\tilde{\nu}_1^{\mathrm{I}} \tilde{\nu}_1^{\mathrm{I}} \to \mathrm{Y_3 Y_4}$	0.1%	
Secluded DM sector	$\tilde{\nu}_1 \tilde{\nu}_1 \to \mathrm{A_s A_s}$	2.9%	7.1%
	$\tilde{\nu}_1 \tilde{\nu}_1 \to \mathrm{h_s h_s}$	0.2%	
	$\tilde{\nu}_1 \tilde{\nu}_1 \to \nu_4 \nu_4$	4.0%	
Higgs portal	$\tilde{\nu}_1 \tilde{\nu}_1 \to \mathrm{h h_s}$	0.1%	6.8%
	$\tilde{\nu}_1 \tilde{\nu}_1 \to \mathrm{hh}$	0.2%	
	$\tilde{\nu}_1 \tilde{\nu}_1 \to \mathrm{t\bar{t}}$	0.1%	
	$\tilde{\nu}_1 \tilde{\nu}_1 \to \mathrm{b\bar{b}}$	1.1%	
	$\tilde{\nu}_1 \tilde{\nu}_1 \to \mathrm{W^+W^-}$	5.4%	

Table 5: The annihilation mechanisms and channels in CP-even heavy h_s scenario, where $\chi_2 \equiv \{\tilde{\chi}_1^0, \tilde{\chi}_1^{\pm}\}$, $\mathrm{Y_1 Y_2} \equiv \{\mathrm{hA_s}, \nu_4\nu_4, \nu_5\nu_5\}$, and $\mathrm{Y_3 Y_4} \equiv \{\mathrm{A_s A_s}, \mathrm{W^+W^-}, \nu_4\nu_4\}$.

Type-I NMSSM

1. Singlet-dominated particles, $\tilde{\nu}_1^0$, h_s, A_s, and ν_h, may form a secluded DM sector. λ_ν, κ and v_s play an crucial role in determining the abundance.
2. Due to limited theoretical framework, DM prefers to co-annihilate with Higgsino-dominated $\tilde{\chi}_1^0$ to obtain the measured abundance.
3. Since constraints from DM experiments on electroweakinos and sleptons are weak, the theory can readily explain the muon g-2. For most cases, the Higgsino-dominated $\tilde{\chi}_1^0$ appears as missing track at the LHC, and can be treated as an effective DM candidate. In this case, constraints from the LHC search for SUSY is weak.

The status of Type-I NMSSM:

Model	MSSM	Z_3-NMSSM		GNMSSM	Type-I NMSSM	B-L NMSSM
DM candidate	Bino	Bino	Singlino	Singlino	Sneutrino	Bino, Singlino, BLino, Bileptino, and sneutrino
DM physics	×	×	×	✓	✓✗	✓
LHC and Δa_μ	×	×	×	✓	✓	✓
EWSB	×	×	✓	✓	✓	✓
Neutrino	×	×	×	×	✓✗	✓
Higgs mass	×	×	×	×	✓✗	✓

Part Seven

Example V: B-L NMSSM

B-L NMSSM

- Vector Superfields

SF	Spin $\frac{1}{2}$	Spin 1	$SU(N)$	Coupling	Name
$\hat{B}$	$\lambda_{\tilde{B}}$	B	$U(1)$	g_1	hypercharge
$\hat{W}$	$\lambda_{\tilde{W}}$	W	SU(2)	g_2	left
$\hat{g}$	$\lambda_{\tilde{g}}$	g	SU(3)	g_3	color
$\hat{B}'$	$\lambda_{\tilde{B}'}$	B'	$U(1)$	g_B	$B-L$

- Chiral Superfields

SF	Spin 0	Spin $\frac{1}{2}$	Generations	$(U(1) \otimes \mathrm{SU}(2) \otimes \mathrm{SU}(3) \otimes U(1))$
$\hat{q}$	$\tilde{q}$	q	3	$(\frac{1}{6}, \mathbf{2}, \mathbf{3}, \frac{1}{6})$
$\hat{l}$	$\tilde{l}$	l	3	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1}, -\frac{1}{2})$
$\hat{H}_d$	H_d	$\tilde{H}_d$	1	$(-\frac{1}{2}, \mathbf{2}, \mathbf{1}, 0)$
$\hat{H}_u$	H_u	$\tilde{H}_u$	1	$(\frac{1}{2}, \mathbf{2}, \mathbf{1}, 0)$
$\hat{d}$	$\tilde{d}_R^*$	d_R^*	3	$(\frac{1}{3}, \mathbf{1}, \overline{\mathbf{3}}, -\frac{1}{6})$
$\hat{u}$	$\tilde{u}_R^*$	u_R^*	3	$(-\frac{2}{3}, \mathbf{1}, \overline{\mathbf{3}}, -\frac{1}{6})$
$\hat{e}$	$\tilde{e}_R^*$	e_R^*	3	$(1, \mathbf{1}, \mathbf{1}, \frac{1}{2})$
$\hat{S}$	S	$\tilde{S}$	1	$(0, \mathbf{1}, \mathbf{1}, 0)$
$\hat{\nu}$	$\tilde{\nu}_R^*$	ν_R^*	3	$(0, \mathbf{1}, \mathbf{1}, \frac{1}{2})$
$\hat{\eta}_1$	η_1	$\tilde{\eta}_1$	1	$(0, \mathbf{1}, \mathbf{1}, -1)$
$\hat{\eta}_2$	η_2	$\tilde{\eta}_2$	1	$(0, \mathbf{1}, \mathbf{1}, 1)$

B-L NMSSM

- Superpotential

$$W_{\mathrm{B-L}} = W_{\mathrm{GNMSSM}} + Y_\nu \hat{\nu} \hat{l} \hat{H}_u + Y_x \hat{\nu} \hat{\eta}_1 \hat{\nu} - \lambda_\eta \hat{s} \hat{\eta}_1 \hat{\eta}_2 + \mu_\eta \hat{\eta}_1 \hat{\eta}_2$$

- Naturally provide a seesaw mechanism for neutrino mass and mixing and the Bileptino mass.
- R-parity is related to gauge symmetry.

B-L NMSSM

- Mass matrix for Neutralinos in the basis $\left(\lambda_{\tilde{B}}, \tilde{W}^0, \tilde{H}_d^0, \tilde{H}_u^0, \lambda_{\tilde{B}'}, \tilde{\eta}_1, \tilde{\eta}_2, \tilde{S}\right)$

$$\begin{pmatrix}
M_1 & 0 & -\frac{1}{2}g_1 v_d & \frac{1}{2}g_1 v_u & M_{BB'} & -g_{BY}v_\eta & g_{BY}v_{\bar{\eta}} & 0 \\
0 & M_2 & \frac{1}{2}g_2 v_d & -\frac{1}{2}g_2 v_u & 0 & 0 & 0 & 0 \\
-\frac{1}{2}g_1 v_d & \frac{1}{2}g_2 v_d & 0 & m_{\tilde{H}_u^0\tilde{H}_d^0} & -\frac{1}{2}g_{YB}v_d & 0 & 0 & -\frac{1}{\sqrt{2}}\lambda v_u \\
\frac{1}{2}g_1 v_u & -\frac{1}{2}g_2 v_u & m_{\tilde{H}_d^0\tilde{H}_u^0} & 0 & \frac{1}{2}g_{YB}v_u & 0 & 0 & -\frac{1}{\sqrt{2}}\lambda v_d \\
M_{BB'} & 0 & -\frac{1}{2}g_{YB}v_d & \frac{1}{2}g_{YB}v_u & M_{BL} & -g_B v_\eta & g_B v_{\bar{\eta}} & 0 \\
-g_{BY}v_\eta & 0 & 0 & 0 & -g_B v_\eta & 0 & m_{\tilde{\eta}_2\tilde{\eta}_1} & m_{\tilde{S}\tilde{\eta}_1} \\
g_{BY}v_{\bar{\eta}} & 0 & 0 & 0 & g_B v_{\bar{\eta}} & m_{\tilde{\eta}_1\tilde{\eta}_2} & 0 & m_{\tilde{S}\tilde{\eta}_2} \\
0 & 0 & -\frac{1}{\sqrt{2}}\lambda v_u & -\frac{1}{\sqrt{2}}\lambda v_d & 0 & m_{\tilde{\eta}_1\tilde{S}} & m_{\tilde{\eta}_2\tilde{S}} & m_{\tilde{S}\tilde{S}}
\end{pmatrix}$$

where

$$m_{\tilde{H}_u^0\tilde{H}_d^0} = -\frac{1}{\sqrt{2}}\lambda v_s - \mu, \quad m_{\tilde{\eta}_1\tilde{\eta}_2} = -\frac{1}{\sqrt{2}}\lambda_\eta v_s + \mu_\eta,$$

$$m_{\tilde{\eta}_1\tilde{S}} = -\frac{1}{\sqrt{2}}\lambda_\eta v_{\bar{\eta}}, \quad m_{\tilde{\eta}_2\tilde{S}} = -\frac{1}{\sqrt{2}}\lambda_\eta v_\eta, \quad m_{\tilde{S}\tilde{S}} = \sqrt{2}\kappa v_s + M_S.$$

Notes

Possible DM candidates:
Bino-, Singlino-, Blino-, Bilepton-dominated neutralino, and sneutrino.
Possible light particles: singlino-dominated Higgs, Bileptonic CP-even Higgs.
Singlet-dominated particles can naturally form a secluded DM sector.

The status of B-L NMSSM:

Model	MSSM	Z_3-NMSSM		GNMSSM	Type-I NMSSM	B-L NMSSM
DM candidate	Bino	Bino	Singlino	Singlino	Sneutrino	Bino, Singlino, BLino, Bileptino, and sneutrino
DM physics	×	×	×	✓	✓✗	✓
LHC and Δa_μ	×	×	×	✓	✓	✓
EWSB	×	×	✓	✓	✓	✓
Neutrino	×	×	×	×	✓✗	✓
Higgs mass	×	×	×	×	✓✗	✓

Part Eight

Conclusions

Conclusion about global fit of supersymmetric theories

1. Experimental data provides many hints to fundamental physics.
2. Global fit deepens greatly our understanding of new physics.
3. Economic supersymmetric theories are facing increasingly strong experimental restrictions, and more complex theory becomes favored to alleviate the constraints.
4. Some seeming independent problems may have a common physical origin. Well motivated theories should be explored in a more sophisticated way.

Summary

Model	Annihilation process	σ^{SI}	σ^{SD}
MSSM	$\tilde\chi_1^0\tilde\chi_i^0, \tilde\chi_1^0\tilde\chi_j^\pm, \tilde\chi_1^0\tilde\ell_k, \tilde\chi_1^0\tilde q_f \to XY$, $\tilde\chi_1^0\tilde\chi_1^0 \to Z^*, H^* \to XY$.	$\propto \left[e\tan\theta_w \frac{m_z}{\mu\left(1-m_{\tilde\chi_1^0}^2/\mu^2\right)} \left(\sin 2\beta + m_{\tilde\chi_1^0}/\mu\right)\right]^2$	$\propto \left[e\tan\theta_w \frac{m_z^2 \cos 2\beta}{2\mu^2\left(1-m_{\tilde\chi_1^0}^2/\mu^2\right)}\right]^2$
Z_3-NMSSM	$\tilde\chi_1^0\tilde\chi_1^0 \to t\bar t$, $\tilde\chi_1^0\tilde\chi_i^0, \tilde\chi_1^0\tilde\chi_j^\pm, \to XY$.	$\propto \left[\frac{\lambda^2 v}{\mu_{\mathrm{eff}}} \frac{1}{1-m_{\tilde\chi_1^0}^2/\mu_{\mathrm{eff}}^2} \left(\frac{m_{\tilde\chi_1^0}}{\mu_{\mathrm{eff}}} - \sin 2\beta\right)\right]^2$	$\propto \left[\frac{\lambda^2 v}{\mu_{\mathrm{eff}}^2} \frac{m_z \cos 2\beta}{1-m_{\tilde\chi_1^0}^2/\mu_{\mathrm{eff}}^2}\right]^2$
GNMSSM	$\tilde\chi_1^0\tilde\chi_1^0 \to h_s A_s$	$\propto \left[\frac{\lambda^2 v}{\mu_{\mathrm{tot}}} \frac{1}{1-m_{\tilde\chi_1^0}^2/\mu_{\mathrm{tot}}^2} \left(\frac{m_{\tilde\chi_1^0}}{\mu_{\mathrm{tot}}} - \sin 2\beta\right)\right]^2$	$\propto \left[\frac{\lambda^2 v}{\mu_{\mathrm{tot}}^2} \frac{m_z \cos 2\beta}{1-m_{\tilde\chi_1^0}^2/\mu_{\mathrm{tot}}^2}\right]^2$
Type-I NMSSM	$\tilde\nu_1\tilde\chi_i^0 \to XY$, $\tilde\nu_1\tilde\nu_1 \to h_s \to XY$, $\tilde\nu_1\tilde\nu_1 \to h_s h_s, A_s A_s, \nu_h \nu_h$.	$\propto \left[-\frac{\sqrt{2}}{\lambda}\left(2\lambda_\nu^2 + \kappa\lambda_\nu\right)\mu_{\mathrm{eff}} + \frac{\lambda_\nu A_{\lambda_\nu}}{\sqrt{2}}\right]^2$	-

Summary

Obtained by both analytic formulae and global fits, which are tough tasks.

Model	MSSM	Z_3-NMSSM		GNMSSM	Type-I NMSSM	B-L NMSSM
DM component	Bino	Bino	Singlino	Singlino	Sneutrino	Bino, Singlino, BLino, Bileptino, or sneutrino
DM physics	×	×	×	✓	✓✗	✓
LHC and Δa_μ	×	×	×	✓	✓	✓
EWSB	×	×	✓	✓	✓	✓
Neutrino	×	×	×	×	✓✗	✓
Higgs mass	×	×	×	×	✓✗	✓

thanks!